An Application of the Simulated Annealing to the Calculation of a Combined Mode Choice and Route Choice Network Equilibrium Model with Road Travel Time Uncertainty

An application of the simulated annealing to the calculation of a combined mode choice and route choice network equilibrium model with road travel time uncertainty

Kazuki Nagao[1)], Shoichiro Nakayama[2)], Jun-ichi Takayama[2)], and Takuya Maruyama[3)]
[1]Oriental Consultants Co., LTD.
OC Sendai Building 104 Tsuchitoi Wakabayashi Sendai 984-0665, Japan
(nagao-kz@oriconsul.co.jp)
[2]Kanazawa University
Kakuma-machi, Kanazawa 920-1192, Japan
[3]Kumamoto University
2-3-1 Kurokami, Kumamoto 860-8555, Japan

Abstract— **Travel time of public transport such as railway and Light Rail Transit (LRT) is more reliable than road traffic. When assessing the effect of railway transits, the reliability should be considered. Otherwise, the effect may be underestimated. Then, we should not only treat both transit and road users simultaneously and consistently, but also travel time uncertainty of cars and the reliability (or punctuality) of railway for exact evaluation. A network equilibrium model with combined mode and route choice considering road travel time uncertainty does not necessarily have a unique solution, but may have many local solutions, because of the interaction between modes. The simulated annealing is one of the useful techniques to such a problem, to the calculation of the network equilibrium model with combined mode and route choice. In this paper, we apply the network equilibrium model proposed to the Kanazawa urban area using the simulated annealing technique.**

1. Introduction

Travel time of public transport such as railway and Light Rail Transit (LRT) is more reliable than road traffic. When assessing the effect of railway transits, the reliability should be considered. Otherwise, the effect may be underestimated. Therefore, we should not only treat both transit and road users simultaneously and consistently, but also consider travel time uncertainty of cars and the reliability (or punctuality) of railway for exact evaluation.

One of the main causes of uncertainty of road traffic is variation of travel demand. We proposed a stochastic network equilibrium model, where the travel demand is normal-distributed and route flows assigned are also normal-distributed [1]. The model enables us to assess network's uncertainty or travel time reliability. Then, we proposed a network equilibrium model with combined mode and route choice considering road travel time uncertainty based on the stochastic network equilibrium model [2].

The network equilibrium model with combined mode and route choice considering road travel time uncertainty does not necessarily have a unique solution, but may have many local solutions, because of the interaction between modes. One of the problem is how we deal with this problem.

The simulated annealing is one of the useful techniques to such a problem when calculating the network equilibrium model with combined mode and route choice. The simulated annealing is applied to solve the complicated network design problem in many studies, e.g. [3-9].

In this paper, we improve the network equilibrium model with combined mode and route choice considering road travel time uncertainty [2] to solve such a problem. We adopt a solution algorithm using the simulated annealing technique. Then, we apply the model to the Kanazawa urban area using the proposed algorithm.

2. Stochastic Traffic Volumes

In this paper, the travel demand is normal-distributed and route flows assigned are also normal-distributed. Assume that each route flow follows an independent normal distribution and that the variance of the flow is proportional to its mean.

Let Q^{rs} denote the random variable of the demand between OD pair rs, E[Q^{rs}] the expected demand ,and Var[Q^{rs}] the variance of the demand. Here, we assume that Var[Q^{rs}] is η E[Q^{rs}].(η is a positive constant parameter.)

The route flows are expressed as:

$$F_k^{rs} \sim N[\mu_k^{rs},\ \eta\mu_k^{rs}] \tag{1}$$

where F_k^{rs} is the random variable of the flow on Route k between OD pair rs, and $N[\mu_k^{rs}, \eta\mu_k^{rs}]$ is the normal distribution which has the expected route flows μ_k^{rs} and the variance of route flows, $\eta\mu_k^{rs}$.

The flow conservation rule of the model is expressed as:

$$Q^{rs} = \sum_{k\in K^{rs}} F_k^{rs} \qquad \forall r\ \forall s\ \forall k \tag{2}$$

$$\mathrm{E}[Q^{rs}] = \sum_{k\in K^{rs}} \mu_k^{rs} \qquad \mathrm{Var}[Q^{rs}] = \sum_{k\in K^{rs}} (\sigma_k^{rs})^2 \qquad \forall r\ \forall s\ \forall k \tag{3}$$

where $(\sigma_k^{rs})^2$ $(=\eta\mu_k^{rs})$ denotes the variance of the flow on Route k between OD pair rs.

Link flows are expressed as:

$$X_a = \sum_{r\in R}\sum_{s\in S}\sum_{k\in K^{rs}} \delta_{a\,k}^{rs} F_k^{rs} \qquad \forall r\ \forall s\ \forall k \tag{4}$$

$$X_a \sim N\left[\sum_{r\in R}\sum_{s\in S}\sum_{k\in K^{rs}} \delta_{a\,k}^{rs}\mu_k^{rs},\ \eta\sum_{r\in R}\sum_{s\in S}\sum_{k\in K^{rs}} \delta_{a\,k}^{rs}\mu_k^{rs}\right] \qquad \forall r\ \forall s\ \forall k \tag{5}$$

where X_a is the random variable of Link a, R and S are the sets of the origin and destination nodes respectively, K^{rs} is the set of the routes between OD pair rs, and $\delta_{a\,k}^{rs}$ is 1 if Link a is part of Route k; otherwise, 0.

3. Travel Time and Generalized Travel Cost

A. Travel Time of Road Traffic

In this paper, we adopt a BPR-type performance function for calculating travel time, and link travel time can be expressed as $\alpha + \beta x^{\gamma}$, where x is a link volume and α, β, and γ are positive constant parameters. When γ is an integer (usually 4.0 is used), the expected link travel time can be calculated using moment generating functions. A moment generating function, $M(s)$, is defined as E[e^{sX}] . As a property of the moment generating function, E[x^{γ}] = $d^n M_a(s)/ds^n\,\big|_{s=0}$. The expected link travel time is:

$$\mathrm{E}[T_a] = t_{a0} + \alpha \cdot \frac{1}{C_a^{\beta}} \cdot \left.\frac{d^{\beta} M_a(s)}{ds^{\beta}}\right|_{s=0} \tag{6}$$

where

t_{a0}: Free-flow travel time

C_a: Capacity

The expected travel time for Route k between OD pair rs, E[T_k^{rs}] is $\sum_{a\in A} \delta_{a\,k}^{rs}\,\mathrm{E}[T_a]$ where T_a denotes the random variable of the flow on Link a. The variance of Link a travel time ,Var[T_a], is E[$(T_a)^2$]–E[T_a]^2. The variance of travel time for Route k between OD pair rs, Var[T_k^{rs}] is $\sum_{a\in A} \delta_{a\,k}^{rs}\,\mathrm{Var}[T_a]$.

B. Travel Time of Public Transport

Travel time of buses is influenced of road traffic volumes. Assume for simplicity that the bus expected travel time on Link a, E[$T_a^{(bus)}$] and the variance of travel time for Link a, Var[$T_a^{(bus)}$] are:

$$\mathrm{E}[T_a^{(bus)}] = \psi\,\mathrm{E}[T_a] \tag{7}$$

$$\mathrm{Var}[T_a^{(bus)}] = \mathrm{Var}[T_a] \tag{8}$$

where ψ is a positive constant parameter.

The train travel time is constant, and is not influenced of road traffic volumes. Therefore, the variance of train travel time is 0.

C. Effective Travel Time

In this paper, we adopt an effective travel time considering user's risk attitude to the travel time uncertainty. The effective travel time contains "*safety margin*" to avoid lateness.

The effective travel times are expressed as:

$$V_k^{rs,c} = \mathrm{E}[T_k^{rs,c}] + \gamma \,\mathrm{Var}[T_k^{rs,c}] \tag{9}$$

$$V_{rs}^{tran} = \mathrm{E}[T_{rs}^{tran}] + \gamma \,\mathrm{Var}[T_{rs}^{tran}] \tag{10}$$

where

- $V_k^{rs,c}$: The effective travel time on Route k between OD pair rs of a car
- V_{rs}^{tran}: The effective travel time between OD pair rs of public transport
- $\mathrm{E}[T_k^{rs,c}]$: The expected travel time for Route k between OD pair rs of a car
- $\mathrm{E}[T_{rs}^{tran}]$: The expected travel time for Route k between OD pair rs of a public transport
- $\mathrm{Var}[T_k^{rs,c}]$: The variance of travel time for Route k between OD pair rs of a car
- $\mathrm{Var}[T_{rs}^{tran}]$: The variance of travel time for Route k between OD pair rs of a public transport
- γ: A parameter on user's risk attitude ($\gamma > 0$; risk averse, $\gamma = 0$; risk neutral, $\gamma < 0$; risk loving)

D. Generalized Travel Cost

Using the effective travel time, we define a generalized travel time for a network equilibrium model with combined mode. The generalized travel time includes the effective travel time, a fare of a public transport, etc. A user chooses a travel mode and a route based on generalized travel times.

The generalized travel costs are expressed as:

$$c_k^{rs,c} = \tau V_k^{rs,c} + \xi \tag{11}$$

$$c_{rs}^{tran} = \tau (V_{rs}^{tran} + w_{rs} + \iota_{rs}) + m_{rs} \tag{12}$$

where

- $c_k^{rs,c}$: The generalized travel costs for Route k between OD pair rs of a car
- c_{rs}^{tran}: The generalized travel costs between OD pair rs of a public transport
- w_{rs}: The waiting time between OD pair rs of a public transport
- ι_{rs}: The access and egress time
- m_{rs}: The fare between OD pair rs of a public transport
- τ: Value of time
- ξ: A parameter (a maintenance cost for car)

4. Formulation

A. Assumption on user's attitude to choose a travel mode

In order to determine the split rate of travel modes, a logit type model is adopted.

The probability that a user chooses a car as traffic mode is given by:

$$P_{rs}^c = \frac{1}{1 + \exp\{-\theta (c_{rs}^{tran} - \lambda_{rs}^c)\}} \qquad \forall r \ \forall s \tag{13}$$

where

- P_{rs}^c: The probability that a (representative) user chooses between OD pair rs chooses a car
- λ_{rs}^c: The minimum of the generalized travel costs between OD pair rs of a car
- θ: A positive constant parameter

B. Assumption on user's attitude of route choice

In this paper, we adopt on a generalized Wardrop's equilibrium [10] for road traffic.

The generalized Wardrop's equilibrium model is formulated as follows:

$$\mu_k^{rs,c} \left(c_k^{rs,c} - \lambda_{rs}^c \right) = 0 \qquad \forall r \ \forall s \ \forall k \tag{14}$$

$$c_k^{rs,c} \geq \lambda_{rs}^c, \quad \mu_k^{rs,c} \geq 0 \qquad \forall r \ \forall s \ \forall k \tag{15}$$

$$\sum_{k \in K^{rs}} \mu_k^{rs,c} = q_{rs}^c \qquad \forall r \ \forall s \tag{16}$$

where

- $\mu_k^{rs,c}$: An average of the flow on Route k between OD pair rs

$q_{rs}{}^{c}$: An average of the flow between OD pair *rs* of a car

C. Formulation

In this paper, we assume that the choice of travel modes is based on a logit type model and user chooses the route of minimum generalized cost for road traffic. The equilibrium model of this study is formulated as follows:

$$q_{rs}^{c} = \frac{q_{rs}}{1+\exp\{-\theta(c_{rs}^{tran} - \lambda_{rs}^{c})\}} \quad \forall r, \forall s \tag{17}$$

$$q_{rs} = q_{rs}^{c} + q_{rs}^{tran} \quad \forall r, \quad \forall s \tag{18}$$

$$\mu_{k}^{rs,c}\left(c_{k}^{rs,c} - \lambda_{rs}^{c}\right) = 0 \quad \forall r, \forall s, \forall k \tag{14}$$

$$c_{k}^{rs,c} \geq \lambda_{rs}^{c} \quad \forall r, \forall s, \forall k \tag{19}$$

$$\sum_{k \in K^{rs}} \mu_{rs,k}^{c} = q_{rs}^{c} \quad \forall r, \quad \forall s \tag{16}$$

$$q_{rs}^{c}, q_{rs}^{tran} \geq 0 \quad \forall r, \quad \forall s \tag{20}$$

$$\mu_{k}^{rs,c} \geq 0 \quad \forall r, \forall s, \forall k \tag{21}$$

$$c_{k}^{rs,c} = \tau V_{k}^{rs,c} + \xi \quad \forall r, \forall s, \forall k \tag{11}$$

$$c_{rs}^{tran} = \tau(V_{rs}^{tran} + w_{rs} + \iota_{rs}) + m_{rs} \quad \forall r, \quad \forall s \tag{12}$$

$$V_{k}^{rs,c} = \mathrm{E}[T_{k}^{rs}] + \gamma \mathrm{Var}[T_{k}^{rs}] \quad \forall r, \forall s, \forall k \tag{9}$$

$$V_{rs}^{tran} = \psi \mathrm{E}[T_{k}^{rs}] + \gamma \mathrm{Var}[T_{k}^{rs}] \quad \forall r, \forall s, \forall k \tag{22}$$

where

$q_{rs}{}^{tran}$: An average of the flow between OD pair *rs* of a public transport

q_{rs}: An average of the flow between OD pair *rs*

The above equations can be formulated as a non-linear complementary problem:

$$q_{rs}^{c}\left(\lambda_{rs}^{c} + \frac{1}{\theta}\ln q_{rs}^{c} - \kappa_{rs}\right) = 0 \quad \forall r \ \forall s \tag{23}$$

$$\lambda_{rs}^{c} + \frac{1}{\theta}\ln q_{rs}^{c} - \kappa_{rs} \geq 0 \quad \forall r \ \forall s \tag{24}$$

$$q_{rs}^{tran}\left(c_{rs}^{tran} + \frac{1}{\theta}\ln q_{rs}^{tran} - \kappa_{rs}\right) = 0 \quad \forall r \ \forall s \tag{25}$$

$$c_{rs}^{tran} + \frac{1}{\theta}\ln q_{rs}^{tran} - \kappa_{rs} \geq 0 \quad \forall r \ \forall s \tag{26}$$

$$\mu_{rs,k}^{c}\left(c_{rs,k}^{c} - \lambda_{rs}^{c}\right) = 0 \quad \forall r \ \forall s \ \forall k \tag{27}$$

$$c_{rs,k}^{c} - \lambda_{rs}^{c} \geq 0 \quad \forall r \ \forall s \ \forall k \tag{28}$$

$$\lambda_{rs}^{c}\left(\sum_{k}\mu_{rs,k}^{c} - q_{rs}^{c}\right) = 0 \quad \forall r \ \forall s \ \forall k \tag{29}$$

$$\sum_{k}\mu_{rs,k}^{c} - q_{rs}^{c} \geq 0 \quad \forall r \ \forall s \ \forall k \tag{30}$$

$$\kappa_{rs}\left(q_{rs}^{c} + q_{rs}^{tran} - q_{rs}\right) = 0 \tag{31}$$

$$q_{rs}^{c} + q_{rs}^{tran} - q_{rs} \geq 0 \quad \forall r \ \forall s \tag{32}$$

where

$$\kappa_{rs} = \frac{1}{\theta}\ln q_{rs}^{c} - \frac{1}{\theta}\ln\left(\exp\{-\theta\min\left(c_{rs,k}^{c}\right)\} + \exp(-\theta c_{rs}^{t})\right)$$

5. A Solution Algorithm Using the Simulated Annealing Technique

The non-linear complimentary problem in this paper does not necessarily have a unique solution, but may have many local solutions, because of the interaction between modes. Therefore, in this paper, we introduce the simulated annealing.

The simulated annealing [7] was introduced by Kirkpatrick et al.(1983). The procedure simulates the cooling of hot metals to produce a (cold) solid metal.

The equilibrium model is formulated as a non-linear complimentary problem, Eq. (23-32). We reformulate an optimization problem to solve the non-linear complementary problem. Using the Fischer-Burmeister function [11], the optimization function, $L(\mathbf{x})$, is given by:

$$L(\mathbf{x}) := \sum_{rs}\left[q_{rs}^{c} + \left(\min\left(c_{rs,k}^{c}\right) + \frac{1}{\theta}\ln q_{rs}^{c} - \kappa_{rs}\right) - \sqrt{q_{rs}^{tran^2} + \left(\lambda_{rs}^{tran} + \frac{1}{\theta}\ln q_{rs}^{tran} - \kappa_{rs}\right)^2}\right]^2$$

$$+\sum_{rs}\left[q_{rs}^{tran}+\left(\lambda_{rs}^{tran}+\frac{1}{\theta}\ln q_{rs}^{tran}-\kappa_{rs}\right)\right.$$

$$\left.-\sqrt{q_{rs}^{tran^2}+\left(\lambda_{rs}^{tran}+\frac{1}{\theta}\ln q_{rs}^{tran}-\kappa_{rs}\right)^2}\,\right]^2$$

$$+\sum_{rs}\sum_{k}\left[\mu_{rs,k}^{c}+\left(c_{rs,k}^{c}-\min\left(c_{rs,k}^{c}\right)\right)\right.$$

$$\left.-\sqrt{\mu_{rs,k}^{c\ 2}+\left(c_{rs,k}^{c}-\min\left(c_{rs,k}^{c}\right)\right)^2}\,\right]^2 \qquad (33)$$

The algorithm is as follows:

Step 1 : Set up an initial solution ($q_{rs}^{c(1)}$, $q_{rs}^{tran(1)}$, $\mu_k^{rs,c(1)}$).

Step 2 : Update the temporary solution

(a) Calculate the minimum of the generalized travel costs between OD pair rs of a car, $\lambda_{rs}^{c(n)}$.

(b) Calculate the auxiliary variable about expected flows , $\upsilon_{rs}^{(n)}$, $\nu_{rs}^{c(n)}$, using the expression as follows.

$$\upsilon_{rs}^{(n)}=\mathrm{E}[Q^{rs}]\frac{1}{1+\exp[-\theta(\lambda_{rs}^{c(n)}-c_{rs}^{tran(n)})]}$$

$$\nu_{rs}^{c(n)}=\mathrm{E}[Q^{rs}]-\upsilon_{rs}^{(n)}$$

(c) Calculate the auxiliary variable by finding the descent direction about expected link flows, $y_a^{(n)}$ by distributing $\{\nu_{rs}^{(n)}\}$ using all-or-nothing method.

(d) Substitute $\boldsymbol{\mu}^{(n+1)}$, $\mathbf{q}^{c(n+1)}$, and $\mathbf{q}^{tran(n+1)}$ as follows for Z(·) and obtain the temporary step size by minimizing Z(·).

$$\boldsymbol{\mu}^{(n+1)}=\boldsymbol{\mu}^{(n)}+\zeta'^{(n)}(\mathbf{y}^{(n)}-\boldsymbol{\mu}^{(n)})$$
$$\mathbf{q}^{c(n+1)}=\mathbf{q}^{c(n)}+\zeta'^{(n)}(\boldsymbol{\nu}^{(n)}-\mathbf{q}^{c(n)})$$
$$\mathbf{q}^{tran(n+1)}=\mathbf{q}^{tran(n)}+\zeta'^{(n)}(\boldsymbol{\upsilon}^{(n)}-\mathbf{q}^{tran(n)})$$

where

- $\boldsymbol{\mu}$: The vector of expected route flows by a car
- $\mathbf{q}^c$: The vector of expected OD flows by a car
- $\mathbf{q}^{tran}$: The vector of expected OD flows by a public transport
- ζ' : The temporary step size

(e) Obtain the (formal) step size $\zeta^{(n)}$ by the expression as follows using the simulated annealing technique.

$$\zeta^{(n)}=\zeta'^{(n)}+\varepsilon^{(n)}$$

where

- ζ : The (formal) step size
- $\varepsilon^{(n)}$: The normal random number in the nth iteration.

The mean of the normal random number is 0 and the variance decreases as the step goes by. In this study, $\varepsilon^{(n)}$ is $T^0/(1+n)$, where T^0 is the initial variance. The solution is obtained after sufficient iteration.

Step 3 : After convergence, the best solution found throughout this process is the result of the algorithm.

6. Applying to the Kanazawa Urban area

In this section, we apply the network equilibrium model proposed to the Kanazawa urban area using the simulated annealing technique.

Figure 1 shows the network. The network consists of 178 nodes and 489 links. As the value of the parameter about the variance of demand, η , 42.0 is used. As BPR-type cost-flow performance function for calculating travel time, $t_a=t_{a0}[1+0.15(x/C_a)^4]$ is used. As value of time, τ, 40(yen/mimute) is used. As

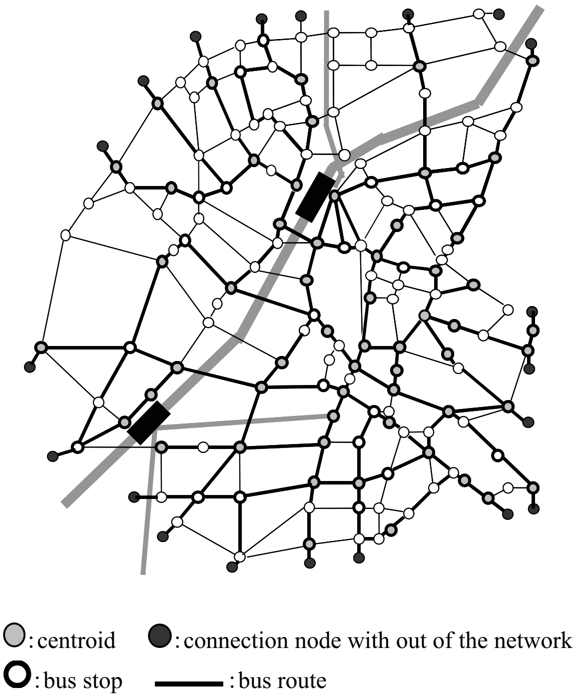

Figure 1. The Kanazawa urban area network

Table 1. Comparison of Algorithms (θ=0.003, ξ=200, γ=1.0, ε=0.01)

	The number of calculation	Correlation coefficient		
		Car	Public transport	All
Using the simulated annealing	30	0.9192	0.2831	0.8780
Not using the simulated annealing	123	0.8975	0.2812	0.8425

the value of the parameter used in the simulated annealing, T^0, 0.01 is used. As the convergence standard of relaxation problem, the maximum of the rate of change of the effective flows, ε, is used.

Table 1 shows the comparison the algorithm using the simulated annealing with not using. In this case, the algorithm using the simulated annealing is quicker at figures than not using. But, the correlation coefficient of a public transport is low value at both of the algorithms.

7. Conclusions

We propose a method for calculating the combined mode and route choice equilibrium model using the simulated annealing. In this paper, we apply the network equilibrium model proposed [2] to the Kanazawa urban area using the simulated annealing technique. As a result of this study, it is found that the simulated annealing is the useful techniques solving a network equilibrium model with combined mode. The future work is to improve the accuracy of the model, to inspect further the advantage of the simulated annealing applying the model to other networks and so on.

References

[1] Nakayama S. Takayama J. Nagao K. and Tokoro T., "Stochastic Network Equilibrium Models for Real Road Network and Their Effect Analysis For Providing Traffic Information," *Journal of Infrastructure Planning and Management (D)*, Vol. 62, No. 4, pp. 526-536 (in Japanese), 2006.

[2] Nagao K.,Nakayama S. Takayama J., "A Combined Network Equilibrium Model Considering Travel Time Uncertainly," *Proceedings of the Infrastructure Planning Committee*, on CD-ROM (in Japanese), 2005.

[3] Friesz Terry L., Cho Hsun-Jung, Mehta Nihal, J., Tobin Roger L. and Anandalingam G., "A Simulated Annealing Approach to the Network Design Problem with Variational Inequality Constraints," *Transportation Science*, Vol.26, No. 1, pp. 18-26, 1992.

[4] Meng Q. and Yang H., "Benefit Distribution and Equity in Road Network Design," *Transportation Research*, Part B 36(1), pp. 19-35, 2002.

[5] Yang H., and Zhang Z. and "Multiclass Network Toll Design Problem with Social and Special Equity Constraints," *Journal of Transportation Engineering*, 128(5), pp. 420-428, 2002.

[6] Cipriani E. and Fusco G. "Solution Procedures for the Global Optimization of Signal Settings and Traffic Assignment Combined Problem," *Traffic and Transportation Studies Proceeding of ICTTS 2002 2*, pp. 833-840, 2002.

[7] Drezner Z. and Wesolowsky G. O., "Network Design: Selection and Design of Links and Facility Location," *Transportation Research*, Part A 37(3), pp. 241-256, 2003.

[8] Wang J. and Wei L., "An ANTS Heuristics for User Equilibrium Assignment," *Proceeding of the Conference on Traffic and Transportation Studies,* ICTTS 4, pp. 533-540, 2004.

[9] Huang Y. –F. and Liu T., "Bi-Level Programming Model and Algorithm of Optimal Toll Rate for Highway Network," *Journal of Traffic and Transportation Engineering*, 6(4), pp. 105-111, 2006.

[10]Wardrop, J.G.: "Some Theoretical Aspects of Road Traffic Research," *Proceedings of the Institute of Civil Engineers Part II*, pp.325-378, 1952

[11]Fischer, A., H.Y. Jiang: "Merit function for complementarity and related problems: A survey," *Computational Optimization and Applications*, Vol. 17, pp. 159-182, 2000.