Synthesis of Planar Multilink Mechanisms Based on Genetic Algorithms*

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In this paper we present a method for synthesizing planar multilink mechanisms which can generate desired planar paths. In previous studies, various optimization methods were applied to synthesize path-generation planar multilink mechanisms. The techniques used, however, were often cumbersome and computationally costly to apply. These problems are mainly due to the high nonlinearity of synthesis mechanisms and the existence of many local solutions. Genetic algorithms have recently been noted as a method for solving the problems described above. Therefore, a new method for synthesizing planar multilink mechanisms is presented based on genetic algorithms. The present method require no initial mechanisms and can search for multiple appropriate mechanisms simultaneously. Furthermore, it can be applied to various synthesis problems. We focus on a closed curve adapting synthesis. As an example, configurations of 4-bar and 6-bar planar mechanisms are determined in a practical application.

Key Words: Link Mechanisms, Syntheses, Genetic Algorithms, Path Generation, Curve Adapting

1. Introduction

Planar multilink mechanisms are often used in automatic machines and robots, because they can make complicated curves and functions in spite of their simplicity[1]. As a result, many methods for synthesizing multilink mechanisms have been investigated[2]-[4].

There are generally two types for synthesizing planar multilink mechanisms. The first, called the synthesis of path generation mechanisms, synthesizes mechanisms, such that coupler points of multilink mechanisms trace desired paths. The second, called the synthesis of function generation mechanisms, synthesizes mechanisms, such that relations between crank angle and follower angle displacements satisfy the desired functional values. Synthesis of path generation mechanisms is more difficult than that of function generation mechanisms[5]. The most representative method for synthesizing path generation planar multilink mechanisms is to determine mechanism dimensions by specifying multiple precision points through which the coupler points should pass[3]. Although the accuracy of multilink mechanisms improves if the number of specified precision points increases, the number of high-degree simultaneous equations to be solved also increases markedly[2]. Therefore, the appropriate mechanisms would not be found easily. Furthermore, it is difficult to keep generation curves of mechanisms the same as given continuous curves, because the accuracy between precision points is not guaranteed. However, there are many kinds of machines that utilize the paths of coupler curves of planar multilink mechanisms such as transport machines[6] and walking robots[7], so that development of synthesis methods for path generation mechanisms is required.

The synthesis of planar multilink mechanisms of which the coupler curves match desired continuous curves is called curve matching synthesis in this paper. Many such methods are under investigation. Most of them utilize optimization methods based on the nonlinear least squares method[8] to minimize

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Table 1 Expression method for 4-bar planar mechanism

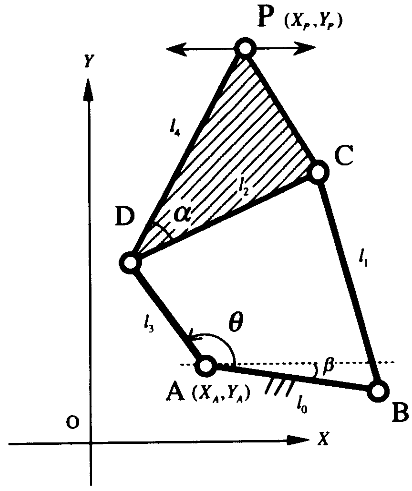
Design parameter	l_0	l_1	l_2	l_3	l_4	α	β	X_A	Y_A
Value	200.0 mm	250.0 mm	250.0 mm	100.0 mm	320.0 mm	30.0°	15.0°	15.0 mm	22.0 mm
Phenotype	2000	2500	2500	1000	3200	3000	1500	0150	0220
Genotype	001··00	010··00	010··00	000··00	011··00	010··00	001··00	000··10	000··00
Chromosome type	001··00 010··00 010··00 000··00 011··00 010··00 001··00 000··10 000··00								

errors between desired paths and generation curves, so that mechanism dimensions should be determined[(5),(9)-(11)]. Furthermore, synthesis methods based on a neural network[(12)] and on the effective curve matching method[(13)] were proposed recently. However, most of them are faced with the problems of cumbersome computations and careful choice of initial mechanisms[(2),(9)]. A synthesis method in which mechanisms satisfying desired conditions are searched for by means of random numbers had been proposed for its simplicity[(11),(14)]. However, it is seldom used because suitable solutions are not always obtained[(2)].

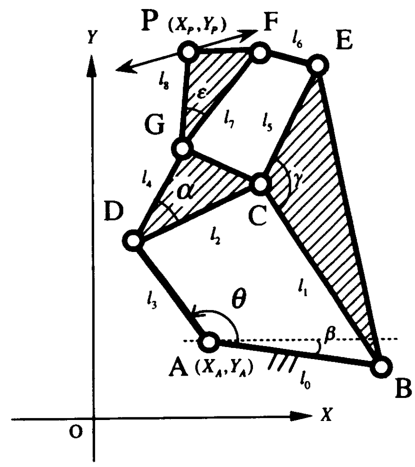
The reasons why it is difficult to apply such optimization methods to the synthesis of path generation mechanisms are the strong nonlinear relation between design variables of mechanisms and coupler curve shapes, and the existence of multiple solutions.

Recently, attention has been paid to genetic algorithms (GA) as a means of solving the above problems effectively[(15),(16)]. GA is an optimization method that imitates the biological evolution process on a computer; thus it is suitable for solving problems with strong nonlinearity or problems that have multiple solutions[(15),(16)]. Namely, GA is an effective method for the synthesis of planar multilink mechanisms.

We apply GA to the curve matching synthesis of planar multilink mechanisms. The synthesis method proposed here can be used widely, and does not need initial data so that desired mechanisms can be obtained with simple computation. We also synthesize some multilink mechanisms in practice to reveal the utility of this method. Here, the curve matching synthesis that uses a closed curve as a desired curve is investigated.



(a) 4-bar planar mechanism



(b) 6-bar planar mechanism

Fig. 1 Planar multilink mechanism

2. Synthesis Method

2.1 Phenotype and genotype of planar multilink mechanisms

A 4-bar and a 6-bar planar mechanism are shown in Figs. 1(a) and (b), respectively. Links AD and AB in Fig. 1(a) are a crank and a fixed link. The 6-bar planar mechanisms are roughly classified into several types[(17)]. The mechanism shown in Fig. 1(b)[(18)], of which the angle displacement relations between crank and follower are the same as those of 4-bar planar mechanisms so that movement analysis is easy, is used here.

It is necessary to express design variables such as link length and link angle of a mechanism by means of introducing genotypes when GA is applied. The expression method for the 4-bar planar mechanism is shown in Table 1. The design variables that are expressed by integers of four figures are defined as phenotype and those that are expressed by binary numbers of 13 figures are defined as genotype G_i. One planar multilink mechanism is defined by one group of genotypes $G_0 \sim G_n$ (n is the number of design variables). One group of genotypes is called an individual in this paper. Furthermore, the connection of some genotypes comprising an individual as shown in Table 1 is called a chromosome in this paper.

2.2 Process of synthesis

A flowchart of the synthesis is shown in Fig. 2. The GA used in this paper is based on a simple GA[19], which is the most basic in principle. First, initial individuals are generated using random numbers, and then a series treatment of selection, reproduction, crossover and mutation is repeated to obtain solutions. Generally, one series treatment is called one generation, repeated calculation is called generation replacement and the number of calculation repeating times is called generation number. Because specific procedures concerning GA have been introduced in many reports[16], the detail of it is omitted.

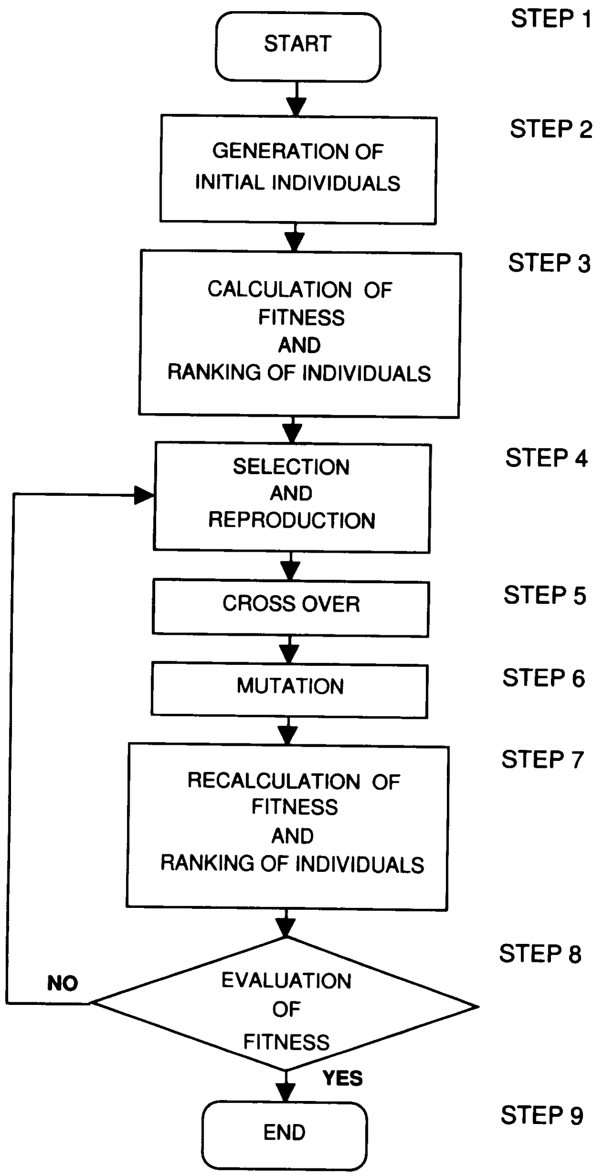


Fig. 2 Flowchart of the synthesis

The operation of each step in Fig. 2 is as follows.

Step 1 (Start) : Calculation conditions such as the population number, rate of selection and reproduction, crossover rate, mutation rate, and the maximum generation number are set. These parameters affect the computation process of steps 2~8.

Step 2 (Generation) : Individual groups of which the population number is set in step 1 are initialized using random numbers.

Step 3 (Calculation of fitness) : The fitness of each individual, i.e., the planar multilink mechanism, is calculated by means of the calculation method described below.

Step 4 (Selection and reproduction) : All individuals are arranged in order of fitness and then a certain number of individuals whose fitness is low are replaced by the same number of individuals whose fitness is high. The ratio of the number of replaced individuals to the population number is called the rate of selection and reproduction.

Step 5 (Crossover) : Some pairs, each of which consists of two individuals, are selected from among all individuals and crossover is performed on each of them. The ratio of the number of individuals that underwent crossover to the population number is called the crossover rate.

Two crossover methods are used in this synthesis. Similarly to the case of simple GA, normal crossover performed on half of the selected pairs. The normal crossover selects a cross position for each genotype of the individuals using random numbers. The remaining pairs are subjected to blend crossover. Blend crossover[20] is often used when continuous values are expressed by genotype, and the middle value of parent genotype is set as a descendant genotype. In this synthesis, the average values of corresponding design variables of individuals selected from among the pairs are taken as a new genotype. Then, two identical individuals are generated from one pair by means of blend crossover in order to keep the population number constant.

Step 6 (Mutation) : Mutation is performed at only some specific generations selected using random numbers. However, mutation is performed forcedly if the all individuals are the same through twice generations, so as to enable searching for other solutions. The ratio of the number of generations that undergo mutation to the maximum generation number is called the mutation operated rate, the ratio of the number of mutated individuals to the population number is called mutation rate, and the mutation performed forcedly is called forced mutation. The value of any position of the chromosomes of the individual which is taken as a mutation object changes from 0 to 1 and vice versa.

Individuals consisting of a genotype that is not necessarily generated by crossover can be generated by performing mutation as described above, and research areas of solutions can be widened. Meanwhile, it is possible that the genotype of individuals with high fitness will be destroyed[21]. In this synthesis, the number of selecting the left 3 figures of genotype as mutation position is about two times as many as that of the right 10 figures, so that design variables change gently upon mutation. Furthermore, the efficiency for obtaining local optimization solutions, which is shortage of GA[22], can be improved, since the lower figures of design variables will change more frequently using this method.

Step 7 (Recalculation of fitness) : The fitness of all individuals is calculated in the same manner as in step 3. The convergence conditions of solutions are affected greatly by the method used to calculate the fitness. Details of the method for calculating fitness used in this paper will be introduced in next section.

Step 8 (Evaluation of fitness) : The fitness average and the best fitness of all individuals are evaluated from the results of step 7. The synthesis is finished when the above results satisfy certain conditions or the number of repeated generations reaches the maximum generation number. Otherwise, the synthesis will be continued starting from step 4 (Selection and reproduction).

2.3 Methods for calculating fitness

The methods for calculating fitness in steps 3 and 7 shown in Fig. 2 are introduced. First, a desired curve that consists of N scattered points is given. The length and planar inclination of the desired curve are arbitrary. An orthogonal coordinate system is selected at random and the centroid position of the desired curve is determined while the total length of the desired curve is converted to the nondimensional unit length 1. Next, a nondimensional polar coordinate system of which the origin is set on the centroid position is selected and the coordinates (r_i^l, θ_i^l) of the points forming the desired curve in this coordinate system are evaluated.

Concerning a genetic curve, the crank angle of a generated mechanism is changed from 0° to 360° at 2° intervals to evaluate the coordinates of the points forming the curve on the orthogonal coordinate system shown in Fig. 1 and the total length of the curve is converted to the nondimensional unit length 1, similarly to the case for a desired curve. The centroid position of the generated curve in the nondimensional coordinate system is determined, the nondimensional polar coordinate system o-$r\theta$ of which the origin is the centroid position of the curve is selected, and the coordinates (r_i^C, θ_i^C) of the points forming the generated curve are evaluated.

Furthermore, the radius coordinates $r_i^L (i=1\sim180)$ of the desired curve corresponding to the circle coordinates $\theta_i^C (i=1\sim180)$ of the points forming the generated curve are evaluated in the same manner as that in the coordinate system o-$r\theta$. Then, the coordinates r_i^L are linearly approximated using evaluated coordinates (r_i^l, θ_i^l) of the desired curve. The residual sum of squares F^r, which expresses the errors between the desired curve and the generated curve, is calculated using the following formula based on r_i^L and r_i^C. In this paper, F^r represents fitness.

$$F^r=\sum_{i=1}^{N}(r_i^L-r_i^C)^2 \tag{1}$$

In this paper, desired curves are assumed to have no tubercle point and their centroid position is located within a closed curve. Furthermore, it is more desirable that desired curves are smooth and similar to a circle.

The treatment used to evaluate the errors between the curves introduced above is called "curve comparing" in this paper.

3. Examples of Mechanism Synthesis

3.1 Synthesis of 4-bar planar mechanisms

Examples of the synthesis of 4-bar planar mechanisms are described in the following. As shown in Fig. 1(a), $l_i (i=0\sim4)$ is the link length of a 4-bar mechanism, α is the angle that couplers make and β is the angle between a fixed link and a horizontal axis. The fixed link length l_0 and the coordinates of point A are always set as 2 000 and (0, 0), respectively, in the case of phenotype. Those are expressed by the method shown in Table 1 and executed according to the procedure discussed in section 3. All 4-bar planar mechanisms investigated in this paper are crank-rocker mechanisms and every link length $l_i (i=0\sim4)$ satisfies the following conditions[23].

$$\left.\begin{aligned} l_0+l_2&>l_1+l_3\\ l_0&>l_2\\ l_3&<l_1\\ l_0-l_2&<l_1-l_3 \end{aligned}\right\} \tag{2}$$

The length of every link is assumed to be longer than 0. Furthermore, the ratio of crank length to fixed link length l_3/l_0 is assumed to be 0.1 or greater to prevent the total length of a generated curve from becoming much smaller than the link length. The mechanisms that do not satisfy the above conditions are replaced with the mechanisms that are generated again in order to keep the population number set in step 1 constant.

The syntheses were executed using the solid curve in Fig. 3 that was used by Watanabe et al.[13], and the solid curve in Fig. 4 that contains two approximate straight lines intersecting each other. The first curve is called curve 1 and the second is called curve 2. Curve 1 indicated by Watanabe et al. is formed by connecting the arcs of radii 1/3, 1 and 1/3 with central angle $2\pi/3$ and the line of length $2/\sqrt{3}$ as shown in Fig. 5[13]. The dimensions of the 4-bar planar mechanism for the generated curve shown in Fig. 4 are shown in Table 2.

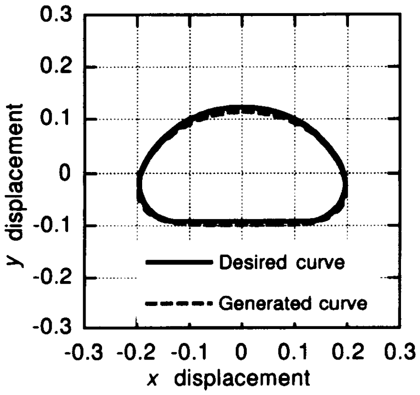


Fig. 3 Curve 1 (Desired and generated curves)

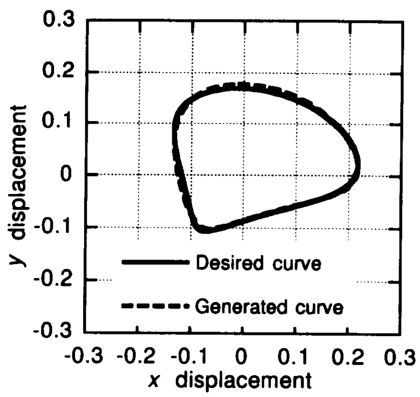


Fig. 4 Curve 2 (Desired and generated curves)

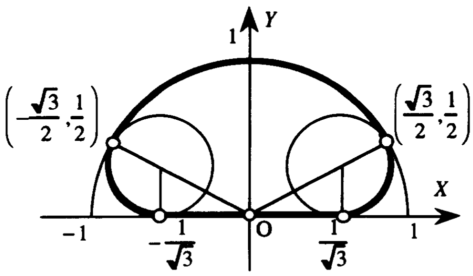


Fig. 5 Desired curve (Curve 1)[13]

Table 2 Dimensions of mechanism for generated curve 2

l_0	l_1	l_2	l_3	l_4	α	β
1.0	2.3	2.5	2.0	1.6	33.8°	0.0°

The values of population, reproduction rate, crossover rate, mutation operation rate and mutation rate shown in Table 3 were used and the forced mutation rate described above was set as 0.5. In this example of the synthesis, stop conditions were judged in terms of the maximum generation number only, without considering the fitness values in order to investigate the convergence conditions of the synthesis. It is judged that the solutions converge if the value of best fitness does not exceed 20×10^{-3}, when calculation is repeated until the maximum generation number. Furthermore the mechanism that can generate the nearly desired curve is regarded to be synthesized if the ratio of the maximum errors between non-dimensional generated and desired curves does not exceed 3%.

The obtained examples of each mechanism are shown in Figs. 6 and 7, and generated curves of these mechanisms are shown by dotted lines in Figs. 3 and 4, respectively. These results shown that obtained curves are similar to the desired curves. The fitness of generated curves and the dimensions of the mechanisms shown in Figs. 6 and 7 are shown in Tables 4(a) and 5(a), respectively. Each link length of the obtained mechanism is expressed in such a way that the fixed link length l_0 is a unit length 1, and the values of fitness are expressed in such a way that the obtained curve is a unit length 1. Figures 8 and 9 show changes of fitness average and best fitness with generation, when curves 1 and 2 were used as desired curves, respectively. Both of them converge at about the 50th generation. There are some generations whose fitness averages change rapidly even after fitness converges. This is because forced mutation is performed in step 6.

The calculation was executed using a Hewlett-Packard model 715/33 system 700 HP-Apollo 9000, and it took about 30 minutes to calculate the 150th generation. The calculation will take about 10 minutes if the fitness of the curve is required to reach the

Table 3 Parameters for each execution process

Population	Reproduction rate	Cross over rate	Mutation operation rate	Mutation rate
500	0.05	0.3	0.2	0.2 (0.5)*

*Forced mutation rate

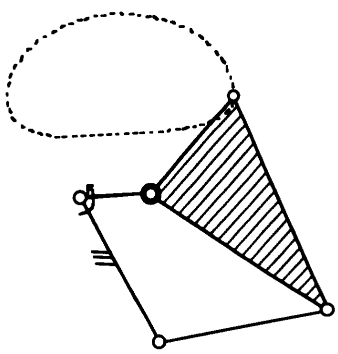


Fig. 6 Obtained 4-bar planar mechanism (Curve 1)

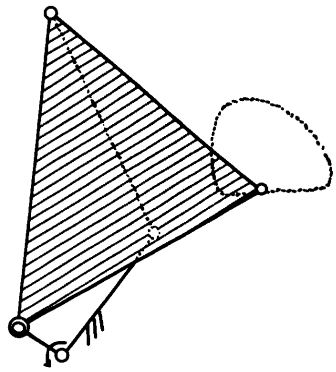


Fig. 7 Obtained 4-bar planar mechanism (Curve 2)

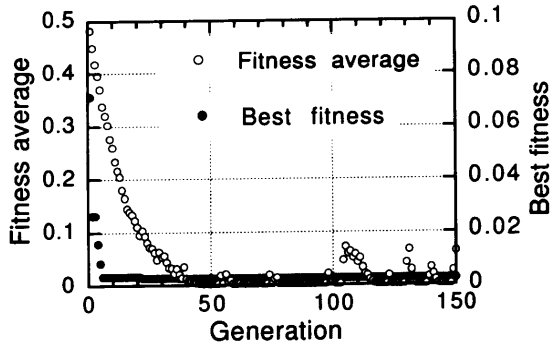


Fig. 8 Convergence condition for synthesizing 4-bar planar mechanism (Curve 1)

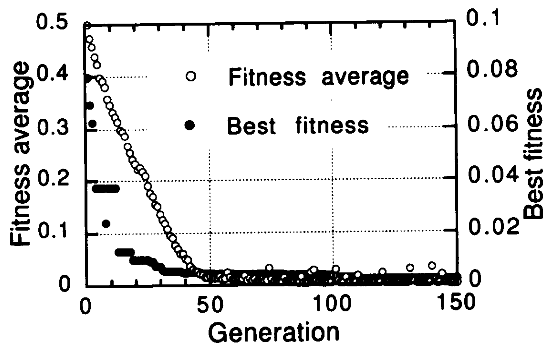


Fig. 9 Convergence condition for synthesizing 4-bar planar mechanism (Curve 2)

level shown in Fig. 3 or 4 in this synthesis.

Furthermore, when the same calculations were performed five times, the mechanism which satisfies the above judgement conditions could be obtained all five times in the case of using curve 1 and four times in the case of using curve 2. Examples of obtained mechanism dimensions and fitness are shown in Tables 4 and 5. The obtained mechanisms described in Table 4(a) and (c) considerably differ from that described in Table 4(b), and in Table 5 the obtained mechanisms differ from each other. Namely, it is possible to obtain multiple mechanisms without considering initial values by using this synthesis, in contrast to the synthesis methods in which other optimization methods are used. The properties of automatic machines and robots can be improved easily if the mechanism with improved properties is selected from among the multiple mechanisms obtained.

Table 4 Dimensions of obtained 4-bar planar mechanisms (Curve 1)

	l_0	l_1	l_2	l_3	l_4	α	β	$F' \times 10^{-3}$
(a)	1.00	1.01	1.27	0.42	0.76	84°	295°	2.73
(b)	1.00	3.44	3.84	0.36	0.81	106°	289°	2.74
(c)	1.00	1.00	1.22	0.47	0.71	79°	294°	4.52

Table 5 Dimensions of obtained 4-bar planar mechanisms (Curve 2)

	l_0	l_1	l_2	l_3	l_4	α	β	$F' \times 10^{-3}$
(a)	1.00	1.59	2.05	0.36	1.86	304°	67°	2.16
(b)	1.00	3.26	2.72	0.32	1.34	291°	356°	2.90
(c)	1.00	0.63	1.04	0.39	1.86	344°	16°	10.8

Although convergence condition changes with the shape of desired curves, the 4-bar planar mechanisms that satisfy the above conditions have been obtained using other desired curves that match the conditions described in section 2. 3. Therefore, the utility of the synthesis has been confirmed.

3. 2 Synthesis of 6-bar planar mechanisms

Although there are only slightly more pairs of 6-bar mechanisms than those of 4-bar mechanisms, 6-bar mechanisms can generate more complicated movement so that their practical value is high[18]. However, the synthesis of 6-bar mechanisms that generate a desired movement is more difficult because of the increase in the number of design variables compared with 4-bar mechanisms[18]. In this paper, we show that the method above described can also synthesize 6-bar planar mechanisms and furthermore that the method can be used widely. As shown in Fig. 1(b) concerning design variables, $l_0 \sim l_8$ are link lengths, α, γ and ε are angles formed by couplers and β is the angle between a fixed link and a horizontal axis. Similar to the case of 4-bar planar mechanisms, the fixed link length l_0 is set as 2 000 and the coordinates of point A are set as (0, 0) in the case of phenotype. The above design variables are expressed in terms of genotype and synthesis is executed using the calculation method of fitness shown in the flowchart in Fig. 2 and section 2. 3. Then it is assumed that each link length of the closed loop part passing through paired points A, B, C and D satisfies the condition of Eq.(2)[17] and is larger than 0, and l_3/l_0 exceeds 0.1. If generated individuals are not crank-rocker mechanisms, the mechanisms are replaced with other generated mechanisms, similarly to that discussed in section 3. 1.

Similar to the case of 4-bar planar mechanisms, the syntheses are executed using solid curves 1 and 2 shown in Figs. 3 and 4 as desired curves. The parameters of the synthesis are shown in Table 3 and only the maximum generation number is set as a stop condition in order to investigate convergence conditions of solutions. Generated curves of obtained mechanisms are shown in Figs. 10 and 11 with the desired curves. As shown in the figures, curves similar to the desired curves are obtained. The shapes of the obtained mechanisms are shown in Figs. 12, 13, and convergence conditions of the solution are shown in Figs. 14, 15. Figs. 12 and 14 are results when the curve 1 is used as the desired curve, and Figs. 13 and 15 are results when curve 2 is used as the desired curve. The dimensions and fitness of the obtained mechanisms are shown in Table 6. Mechanism dimensions are shown by setting a fixed link length as a unit length and

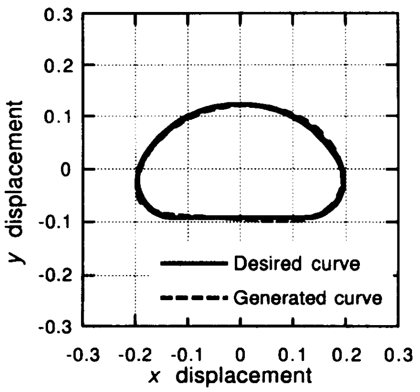


Fig. 10 Curve 1 (Desired and generated curves)

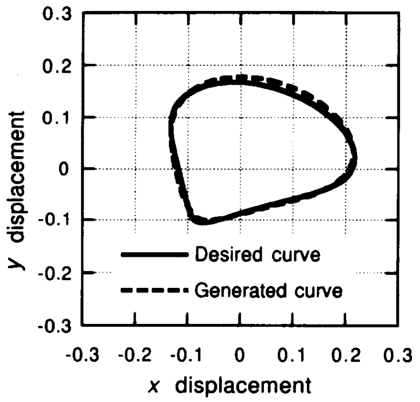


Fig. 11 Curve 2 (Desired and generated curves)

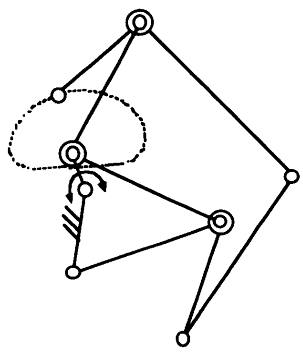


Fig. 12 Obtained 6-bar planar mechanism (Curve 1)

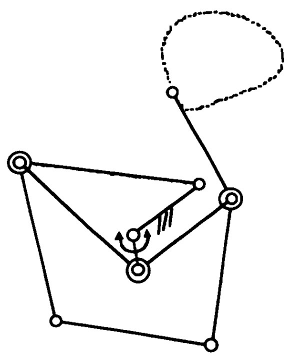


Fig. 13 Obtained 6-bar planar mechanism (Curve 2)

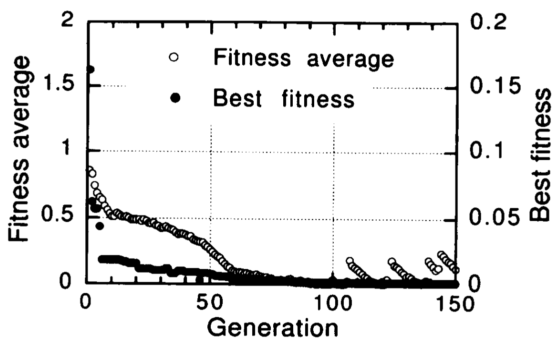


Fig. 14 Convergence condition for synthesizing 6-bar planar mechanism (Curve 1)

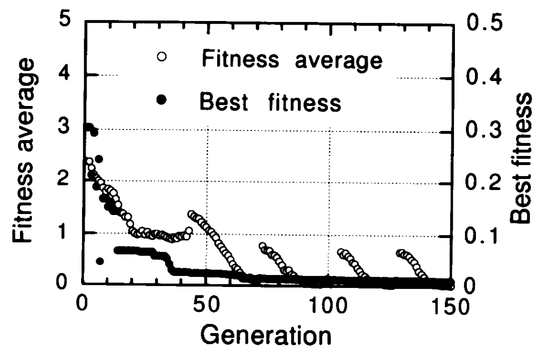


Fig. 15 Convergence condition for synthesizing 6-bar planar mechanism (Curve 2)

Table 6 Dimensions of obtained 6-bar planar mechanisms

	l_0	l_1	l_2	l_3	l_4	l_5	l_6	l_7	α	β	γ	ε	$F' \times 10^{-3}$
Curve 1	1.00	1.83	1.95	0.46	1.73	1.46	2.33	2.52	86.7°	257.6°	53.4°	266.7°	1.20
Curve 2	1.00	2.16	1.91	0.41	1.38	1.90	1.85	1.70	260.2°	32.4°	246.5°	291.2°	11.1

fitness is evaluated by taking the whole length of the curve as the unit length.

The best fitness converges at about the 60th generation as shown in Figs. 14 and 15. It is found that fitness average changes more vigorously during generation evolution by means of mutation operation compared with the case of 4-bar planar mechanisms.

The same calculations for obtaining the solution that satisfies the above judgement conditions were performed five times as the maximum generation number was set at 150; then the solutions were obtained three times when desired curve 1 was used and two times when desired curve 2 was used. It is found that the convergence conditions for synthesizing 6-bar planar mechanisms using this method are worse than those for synthesizing 4-bar planar mechanisms. The search area of solutions using GA rapidly becomes wider with the increase in the number of parameters so that it can be predicted that the search for mechanisms becomes more difficult with the increase of number in the design variables.

The calculation for the synthesis of 6-bar planar mechanisms took about 120 minutes until the 150th generation by means of a FACOM 760/20, FUJITSU. Although the fitness convergence conditions are not necessarily good and the probability of obtaining appropriate solutions is lower than that for 4-bar planar mechanisms, it is satisfactory for synthesizing in practice. Since kinematic analysis for 6-bar planar mechanisms is more complicated, the calculation time becomes longer than that for 4-bar planar mechanisms. Calculation time can be shortened by using more effective kinematic analysis methods.

From the above results, it is confirmed that the present synthesis method is suitable for 6-bar planar mechanisms as well as 4-bar planar mechanisms.

4. Conclusions

The method for synthesizing a path generation planar multilink mechanism based on genetic algorithms, which can be used to specify a continuous closed curve as a desired curve, is proposed and the utility of this method is introduced in this paper. The conclusions obtained in this paper are summarized as follows.

(1) The synthesis of a path generation mechanism using GA and the calculation of fitness based on curve errors were presented.

(2) The 4-bar planar mechanisms that generated curves similar to desired curves could be obtained in a short time by the synthesis method proposed in this paper so that its practical usefulness was confirmed.

(3) The path generation 6-bar planar mechanisms that generated curves to similar to desired curves could be obtained by the proposed method similarly to the synthesis of 4-bar planar mechanisms.

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