H_∞ filtering Convergence and It's Application to SLAM

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Abstract: KF-SLAM(Kalman filter-SLAM) have been used as a popular solution by researchers in many SLAM application. Nevertheless, it shortcomings of assumption for Gaussian noise limited its efficiency and demand researcher to consider better filter and algorithm to achieve a promising result of estimation. In this paper, we proposed one of its family, the H_∞ filter-based SLAM to determine its competency for SLAM problem. Unlike Kalman filter, H_∞ filter able to work in an unknown statistical noise behavior and thus more robust. It rely on a guess that the noise is in bounded energy and does not require a priori knowledge about the system. Therefore, we proposed the H_∞ filter as other available technique to infer the location for both robot and landmarks while simultaneously building the map. From the results of simulation, H_∞ filter produces better outcome than the Kalman filter especially in the linear case estimation. As a result, H_∞ filter may provides another available estimation methods with the capability to ensure and improve estimation for the robotic mapping problem especially in SLAM.

Keywords: SLAM, Estimation, H_∞ filter, Kalman filter

1. INTRODUCTION

SLAM illustrates an application of mobile robot moving through and unknown environment, doing the observation of its surrounding and simultaneously creating a map that it believes from its measurement. A series of influential seminal papers introduced in 1990's such as Smith and Cheeseman et.al[1] gave an impact to the robotic mapping and finally evolve its name to Simultaneous Localization and Mapping problem(SLAM) and also known as Concurrent Mapping and Localization(CML)[2]. See fig.1 for the illustration of SLAM. Between 3 main techniques in SLAM; The Model-based,

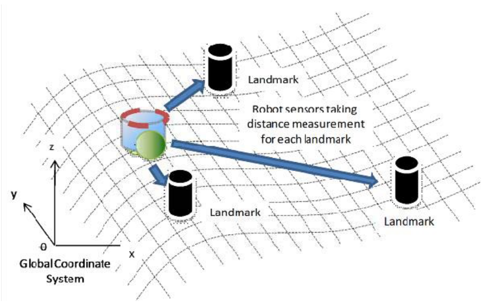

Fig. 1 Illustration for SLAM problem

Behavioural-based and Probabilistic-based SLAM, probabilistic approach has made a significant success as it made less burden than the Model-based approach; which require to build a precise model, or the Behavioral approach; a method of exploiting the sensors application to the system. In spite of probabilistic approach remarkable achievement, there exist shortcomings such as computational complexity. Nevertheless, with modern development of software, a considerable support and solution to this problem may exist, consequently inspire the development of SLAM problem. Kalman filter[3] is one of the most applied algorithm for SLAM. Thanks to its advantages, the application of SLAM has been spread widely whether in outdoor and indoor, outer space, underwater exploration and mining. It also been expanding to 2D and 3D application[4][5][6]. It is expected that the future of SLAM will be likely boosting to home-based application. To realize such expectation, researchers work out in various type of filter, parametric or non-parametric filter to analyze and understand their characteristics and performance. One of the non-parametric filter known as FastSLAM seems to be a convincing available filter among the others to raise the accuracy of estimation. Nonetheless, the attempt of applying this method still encounter some of the common problem in SLAM such as the algorithm complexity and cost of computation. Due to these circumstances, Kalman filter still dominating the research of SLAM. To overcome to the weak assumption of Gaussian noise in Kalman filter, in this paper, we develop H_∞ filter-based SLAM which is more robust. H_∞ filter depend on an assumption that the noises are bounded in certain energy level and does not require a priori knowledge of the system. These criteria should gives a merit to H_∞ filter for SLAM application. Until now, based on our reading, the H_∞ filter is not been applied yet in any of SLAM application. Hence, this paper will be analyzing the performance of H_∞ filter-based SLAM. The analysis is done under assumption that the robot is stationary and observing its surrounding landmarks. The landmarks also defined as stationary point landmarks and assumed to be in a planar world. Further than that, as Kalman filter and H_∞ filter are in the same family, we would like to determine its convergence properties and its behavior towards SLAM to demonstrate each capability and performance between these two filters.

2. SLAM ALGORITHM

Practically in robotics localization and mapping problem, noise are often Non-Gaussian with unknown statistical behavior. One of the Gaussian filters, the celebrated Kalman filter(H_2 filter) inherently become incompetent owing to this fact with the attention to realize the truly autonomous robot behavior such as the Robotic Mapping and localization problem. Due to this disadvantages, H_∞ filter which is also known as minimax filter is proposed in this paper to estimate robot and landmarks location. Having almost the same characteristics as of Kalman filter, H_∞ filter only depend on assumption for a finite uncertainty. These subsequently may provides another available estimation methods with the capability to ensure and improve estimation for the robotic mapping problem especially in SLAM. Throughout this paper, we examine the Kalman filter and H_∞ filter performance in linear case SLAM. We investigate the results using a constant motion and sensors uncertainties with perfect data association. To this extent, H_∞ filter is still not being applied in the SLAM, although it have a desirable properties and competitive compare to that of Kalman filter.

2.1 SLAM Preliminary Model

For the SLAM process model, we have the following[3].

$$x_{v_{k+1}} = F_{v_k} x_{v_k} + u_{v_{k+1}} + v_{v_{k+1}}, \tag{1}$$

where F_{v_k} is the state transition matrix, x_{v_k}, is the vehicle state, u_{v_k} a vector of control inputs, and v_{v_k} a vector of temporally uncorrelated process noise errors with zero mean and covariance, Q_{v_k}. The location of the i^{th} landmark is denoted as p_i. For the stationary landmarks p, and for $i = 1 \dots N$ landmarks,

$$p_{i_{k+1}} = p_{i_k} = p_i \tag{2}$$

On behalf of the measurement process, for an observation at i_{th} landmark/feature,

$$\begin{aligned} z_k &= H_k x_k + w_k & (3)\\ &= H_{p_i} p_i - H_{v_k} x_{(v_k)} + w_k & (4) \end{aligned}$$

where w_k is a vector of temporally uncorrelated observation errors with zero mean and variance R_k. H_k is the observation matrix and represent the output of the sensor z_k to the state vector x_k when observing the i^{th} landmark.

3. H_∞ FILTER CONVERGENCE PROPERTIES

3.1 H_∞ filter-Based SLAM

The papers in [7][8] presents a satisfactory explanation of the H_∞ filtering. Referring to those, we first make an assumption for the noise characteristics same to Kalman filter approach which is zero mean gaussian noise.

Assumption 1: $R \triangleq DD^T > 0$

The above assumption is included to explain that the measurements are correlated with noises. We also make an assumption that the noise is in bounded energy which also a characteristics of H_∞ filter. This is one of the difference between H_∞ filter and Kalman filter.

Assumption 2: Bounded noise energy; $\sum_{t=0}^{N} \|w_k\|^2 < \infty, \sum_{t=0}^{N} \|v_k\|^2 < \infty$

$\Sigma_0 > 0$, $Q_k > 0$, and $R_k > 0$ are the weighting matrices for state x_k, noise w_k, and v_k respectively.

We analyzed the differences between these two filters and found the followings. For Kalman filter, the equation is shown as follow.

$$\begin{aligned} K_k &= P_k(I + H_k^T R_k^{-1} H_k P_k)^{-1} & (5)\\ P_{k+1} &= F_k P_k (I + H_k^T R_k^{-1} H_k P_k)^{-1} F_k^T + Q_k & (6) \end{aligned}$$

whereas H_∞ filter, the equation for its gain and covariance are given by;

$$\begin{aligned} K_k &= P_k(I - \gamma^{-2} P_k + H_k^T R_k^{-1} H_k P_k)^{-1} & (7)\\ P_{k+1} &= F_k P_k (I - \gamma^{-2} P_k + H_k^T R_k^{-1} H_k P_k)^{-1} F_k^T \\ &\quad + Q_k & (8) \end{aligned}$$

From these equations, the dissimilarity between both filter can be explicitly obtained. Furthermore, one should notice that H_∞ filter depends on the covariance matrix of errors signals, Q_k, R_k and L which are chosen and designed to achieve desired performance and all of these parameters must be bigger than zero. Besides that, if γ values becomes bigger, this equation will be the same as of Kalman filter equations.

3.2 Preliminary Results

We begin the convergence analysis of H_∞ filter by defining some assumption as stated below.

Assumption 3: rank F_k=n, for all $k = 0, 1, \dots, N$. Consider that assumption(3) is fulfilled. Then the solution of an H_∞ filtering problem will be as following [7],

$$\begin{aligned} P_{k+1} &= F_k P_k \psi_k^{-1} F_k + G Q_k G^T, \quad P_0 = \Sigma_0 & (9)\\ \psi_k &= I_n + (H_k^T R_k^{-1} H_k - \gamma^{-2} L^T L) P_k & (10) \end{aligned}$$

which holds a positive definite solution if it satisfies an equation below.

$$\hat{P}_k^{-1} - \gamma^{-2} L^T L > 0, \quad t = 0, 1, \dots, N, \tag{11}$$

where

$$\hat{P}_k = (P_k^{-1} - H_k^T R_k^{-1} H_k) > 0 \tag{12}$$

For $\gamma > 0$, the suboptimal H_∞ filter is given by below equations.

$$\begin{aligned} \hat{z}_k^* &= L \hat{x}_{k|k}, \quad \hat{x}_{k+1|k} = F_k \hat{x}_{k|k} & (13)\\ \hat{x}_{k|k} &= \hat{x}_{k|k-1} + K_k [y_k - H_k \hat{x}_{k|k-1}], \hat{x}_{0|-1} = \bar{x}_0 & (14)\\ K_k &= P_k H_k (H_k P_k H_k^T + R_k)^{-1} & (15) \end{aligned}$$

Assumption 4: (F, H) is observable and (F, G) is controllable.

Theorem 1: Assume that Assumptions 1∼3 are satisfied. The map uncertainties will be gradually decreasing as the observations are frequently made by a stationary robot.

Proof: The proof is analogous to [3]. The main difference is the γ effect on the proof. We begin the proof by defining the initial state covariance matrix, $P_0 > 0$ as a positive semidefinite(psd). If the process noise, Q_k and measurement noise, R_k are both psd, for $\gamma > 0$, F, L are all psd, therefore, P_k, ψ_k are also psd. In SLAM, we are interested in finding the estimate location of the robot and the landmarks, therefore, L will be an identity matrix. The Riccati equation for the map covariance matrix and for all landmarks observed, can be defined as

$$\begin{aligned} \det P_{k+1} &= \det[FP_k(I + (H^T R_k^{-1} H - \gamma^{-2} L^T L)P_k)F \\ &\quad + GQ_k G^T] \\ &\leq \det P_k \end{aligned} \tag{16}$$

The rest of the proof is similar to [3] except the differences of the state covariance matrix and the observation matrices which used for the rest of the proof. We also found that for a case of the observation noise, $R >> \gamma$, the covariance matrix will not be a positive definite matrix and therefore overshoot the estimation. Other than that, the whole covariance matrix remain unchanged and stable. ■

Uncertainties are generally relies on the results of the covariance matrix, P. [3] showed convergence properties of Kalman filter-Based SLAM and then analyzed the system behavior based on the Jacobian matrix as an nonlinear version of Kalman filter [9]. For H_∞ filter in linear case, the convergence properties of a stationary robot observing landmarks are still unknown.

Theorem 2: For a stationary robot observing a stationary landmark m, with $\gamma > 0$, as more n-times($n > 0$) observation are made, in the limit, the whole covariance matrix will be converging to

$$P_m^\infty = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{17}$$

Proof: Again we consider a 2D robot with initial covariance matrix P_0, given by the following,

$$P_0 = \begin{bmatrix} \sigma_v^2 & \sigma_{vm} \\ \sigma_{mv} & \sigma_m^2 \end{bmatrix} \tag{18}$$

Assume that, the robot are observing one landmark m. From (13), when the robot is observing m landmarks n times, and for $R = \sigma_r^2$, we will obtain the following.

$$\begin{aligned} \psi &= I + n(H^T R^{-1} H - \gamma^{-2} L^T L)P \\ &= I + \begin{bmatrix} (n\sigma_r^{-2}) - (n\gamma^{-2}) & (n\sigma_r^{-2}) \\ (n\sigma_r^{-2}) & (n\sigma_r^{-2}) - (n\gamma^{-2}) \end{bmatrix} \\ &\quad \begin{bmatrix} \sigma_v^2 & \sigma_{vm} \\ \sigma_{mv} & \sigma_m^2 \end{bmatrix} \\ &= \begin{bmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{bmatrix} \end{aligned} \tag{19}$$

where

$$\begin{aligned} \rho_{11} &= I + [(n\sigma_r^{-2}) - (n\gamma^{-2})]\sigma_v^2 \\ \rho_{12} &= (n\sigma_r^{-2})\sigma_m^2 \\ \rho_{21} &= \sigma_m^2 (n\sigma_r^{-2})\sigma_v^2 \\ \rho_{22} &= I + [(n\sigma_r^{-2}) - (n\gamma^{-2})]\sigma_m^2 \end{aligned}$$

Finding the inverse matrix of (29) using the Matrix Inversion Lemma, yields

$$\psi^{-1} = \begin{bmatrix} \psi_{11} & \psi_{12} \\ \psi_{21} & \psi_{22} \end{bmatrix} \tag{20}$$

where

$$\begin{aligned} \psi_{11} &= [1 + n(\sigma^{-2}\sigma_v^2) - \frac{1}{n^2}\sigma_m^{-2}\sigma_r^2 + \\ &\quad \frac{1}{n}(1 - \gamma^{-2}\sigma_r^{-2})^{-1}\sigma_r^{-2}\sigma_v^2]^{-1} \\ \psi_{12} &= [1 + n(\sigma^{-2}\sigma_m^2) - \frac{1}{n^2}\sigma_m^{-2}\sigma_r^2 + \\ &\quad \frac{1}{n}(1 - \gamma^{-2}\sigma_r^{-2})^{-1}\sigma_r^{-2}\sigma_v^2]^{-1}\sigma_r^{-2}\sigma_m^2 \\ &\quad [n^{-1} + (\sigma^{-2}\sigma_v^2 - \gamma^{-2}\sigma_v^2)]^{-1} \\ \psi_{21} &= [n^{-1} + (\sigma^{-2}\sigma_v^2 - \gamma^{-2}\sigma_v^2)]^{-1}\sigma_r^{-2}\sigma_v^2 \times \\ &\quad [1 + n(\sigma^{-2}\sigma_v^2) - \frac{1}{n^2}\sigma_m^{-2}\sigma_r^2 \\ &\quad + \frac{1}{n}(1 - \gamma^{-2}\sigma_r^{-2})^{-1}\sigma_r^{-2}\sigma_v^2]^{-1} \\ \psi_{22} &= [1 + n(\sigma^{-2}\sigma_v^2 - \gamma^{-2}\sigma_v^2)]^{-1} + [n^{-1} + \\ &\quad (\sigma^{-2}\sigma_v^2 - \gamma^{-2}\sigma_v^2)]^{-1} \times \\ &\quad [1 + n(\sigma^{-2}\sigma_v^2) - \frac{1}{n^2}\sigma_m^{-2}\sigma_r^2 \\ &\quad + \frac{1}{n}(1 - \gamma^{-2}\sigma_r^{-2})^{-1}\sigma_r^{-2}\sigma_v^2]^{-1} \times \\ &\quad \sigma_r^{-2}\sigma_m^2[n^{-1} + (\sigma^{-2}\sigma_v^2 - \gamma^{-2}\sigma_v^2)]^{-1} \end{aligned}$$

From above equation, it can be notice that for a case of the observation noise if the designer must be carefully design the system where for an example of $R >> \gamma$, it cause it to be faulty estimation. The designer must choose an appropriate value to satisfy this condition with considering the theoretical explanation of H_∞ filter. Furthermore, as $n \longrightarrow \infty$,

$$\psi_\infty^{-1} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{21}$$

Substituting this result into (12), finally we obtain that

$$\begin{aligned} P_m^\infty &= FP\psi_\infty^{-1}F \\ &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \end{aligned}$$

■

Corollary 1: For a stationary robot which is observing one landmark, in the limit, the map covariance matrix will be decreasing as follows

$$P_m^\infty = \sigma_m^2 \tag{22}$$

In the limit, the map covariance will be as the following.

$$P_m^\infty = 0 \tag{23}$$

Above results showed better results than obtained by [3]. It is understood that in the limit, the whole covariance matrix is decreasing. Although above result encourages good estimation for the robot and landmark positions, the true landmark location is still unknown[9]. Similar to Kalman filter, if bigger magnitude of noises applied to the filter, the oscillation will become slightly bigger and it consequently effects the overall estimation.

We further observed the convergence properties when the non-moving robot was observing two stationary landmarks.

Theorem 3: For a stationary robot observing two stationary landmarks, m and $\bar{m}$ with $\gamma > 0$, as more $n(n > 0)$ times observation are made, in the limit, the whole covariance matrix will converge to the following,

$$P_m^\infty = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{24}$$

Proof: The proof is similar to Theorem 2 proof. Therefore, it is omitted here. ■

Corollary 2: For a stationary robot observing two landmarks, in the limit, the map covariance matrix remain unchanged as follows;

$$P_m^\infty = \begin{bmatrix} \sigma_m^2 & \sigma_m \sigma_{\bar{m}} \\ \sigma_{\bar{m}} \sigma_m & \sigma_{\bar{m}}^2 \end{bmatrix} \tag{25}$$

In the limit, the map covariance will be as the following.

$$P_m^\infty = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{26}$$

Proof: It is clear from (35) that the map covariance matrix will be shown as in (37). ■

It should be aware that, the map covariance for landmark 1 and landmark 2 will be different and varied to each other especially whenever the initial covariance matrix is not equal to zero and if and only if the uncertainties differ from each other. The process noise and measurement noise must be non-zero to admit these results. As conclusion, the uncertainties of those two landmarks will be differ from each other and thus prove of its dependency to the initial covariance.

4. SIMULATION RESULTS

We demonstrates the simulation results for the above convergence properties for a case of a stationary robot observing two stationary landmarks in an environment of unknown noise but bounded. We show the performance results for a linear case SLAM, in a constant motion and perfect data association as been stated early on this paper. The result of H_∞ filter is being compare to the Kalman filter and H_∞ filter. In the simulation setting, we determine the robot to be located at world coordinate $(1,1)$ while the two landmarks are located with reference to the world coordinate at $(7,7)$ and $(-1,8)$ respectively(see Fig.2).

In order to simplify the analysis, we state the following assumptions.

Assumption 5: Robot is in a planar world.

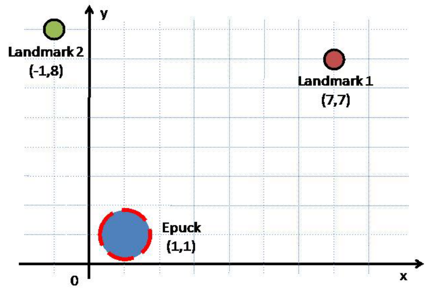

Fig. 2 The global coordinate system representing the location for robot and 2 landmarks to be estimate

Assumption 6: Process error and noise error are small such that both Kalman filter and H_∞ filter are applicable.

Assumption 7: The relative distance between landmarks and robot can be measured.

Assumption 8: Landmarks are assumed to be stationary and consist of point landmarks.

A selection of control parameters for simulation are shown in Table 1. The value of observation noise is set to be less than the γ value as obtained by Theorem 2 and Theorem 3. The location of robot and landmarks are referring to the global coordinate systems.

[htb]

Table 1 SIMULATION PARAMETERS

Process noise, Q	0.0000009
Observation noise,R	0.01
γ	0.9
Robot position	[1,1]
Landmark 1 location	[7,7]
Landmark 2 location	[-1,8]
Initial covariance	$P_{vv} = 0.0000001$, $P_{mm} = 10$

4.1 Stationary robot observing two landmarks

To evaluate H_∞ filter performance, the robot is defined to be more confidence about its location with small uncertainties. We manage the simulation longer about 10000s to determine the stability and consistency between these filters and shows the filter eligibility towards SLAM.

A rate of $\gamma = 0.9$ have been achieved to obtain the best estimation of H_∞ filter. Estimation for the robot location estimation are shown in Figs.3-4 between Kalman filter and H_∞ filter. It is seems that both filter gives their best estimates for each x and y position with a slight difference value. Even though the estimation result both shows similar result, indeed this proves the ability of H_∞ filter in SLAM problem. In addition, the sequel of H_∞ filter promising results continues on the landmarks estimation in Figs.5-6. These figures shown the estimation for both filters on both landmarks. In these figures especially from fig.7, the fast convergence of H_∞ filter is achieved. This is where we believe the distinction between H_∞ filter and Kalman filter. Although it produces very similar result, H_∞ filter improved better convergence. The estimation

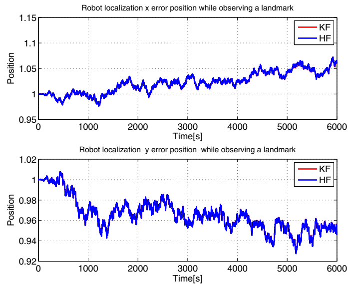

Fig. 3 Robot position estimation between Kalman filter and H_∞ filter

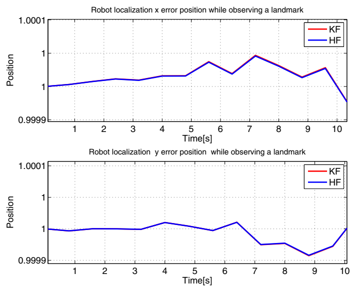

Fig. 4 Convergence comparison between filters

for the landmarks also converging to the true value as it believes. Furthermore, the results also consistent with the result of EKF-based SLAM which also converging to zeros[9]. Even though, it can be see that not much improvement been made by H_∞ filter in the simulation results, the experimental evaluations may encourage better estimation than Kalman filter and presented in later section.

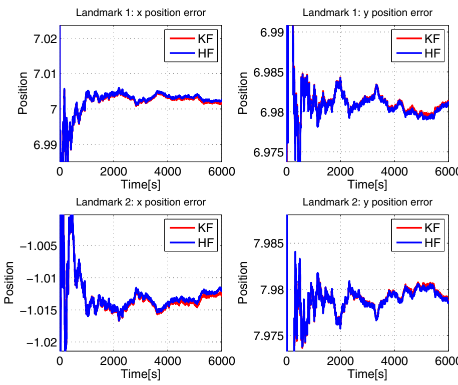

Fig. 5 Landmarks estimation: Kalman filter and H_∞ filter comparison

Conversely, for bigger measurement noise, the estimation of H_∞ filter is diverging and suddenly effecting the competency of H_∞ filter towards estimation. This is the case if the other parameters are remain untouched. See Fig.7 for the effect of bigger observation noise e.g observation noise, $R = 10$. It concludes that, proper design must be carried when adopting H_∞ filter in SLAM to ensure such bad inference can be avoid. Besides, as stated previously, a bigger value of γ will result the approximation to be identical to Kalman filter estimation.

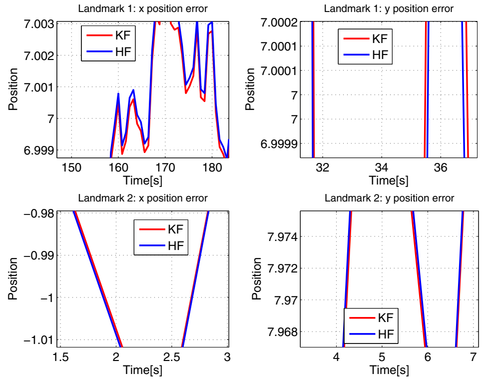

Fig. 6 Landmarks estimation: Kalman filter and H_∞ filter comparison

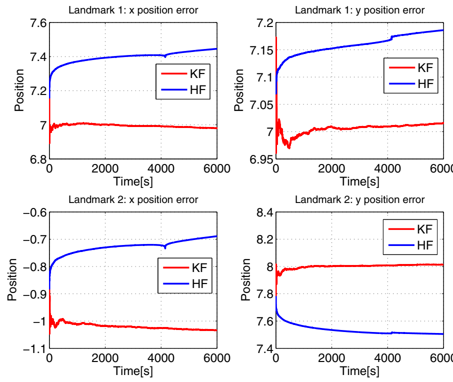

Fig. 7 The landmarks estimation:Effect of bigger measurement noise with unchanged γ

5. EXPERIMENTAL RESULTS

To gain confidence about H_∞ filter, we run an experimental evaluation to understand its behavior in real application. We made the same assumptions for the experiment to ensure that the characteristics and consistency are inherent as shown in the convergence theorems and simulation outcomes. H_∞ filter should perform when $\gamma = 0.65$ and lead to a competent result than Kalman filter. In the experiment, two landmarks are defined at two position with reference to the robot coordinate system at $(50,0)$ and $(60,0)$ in millimeters(mm) respectively. See Fig.8 for experimental setup.

Fig. 8 Stationary Epuck observing landmarks

From Fig.9, Fig.10, and Fig.11, we identify that the H_∞ filter converges faster than Kalman filter. This is considerable result which shows the competency using H_∞ filter. Above mentioned results shows for a case where a robot with minor measurement sensors error or less-

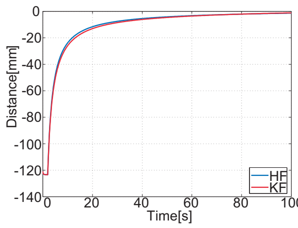

Fig. 9 Robot location estimation: x coordinate

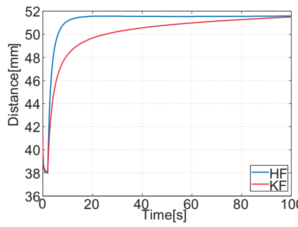

Fig. 10 Landmark 1 Estimation

noisy environment. In contrast, where the when the other parameters are unchanged, but with bigger measurement noise, the H_∞ filter performance is violated and incapable to achieve better results than Kalman filter($Figs.12-13$). Due to this undesirable design, the estimation are faulty and inherently causing undesirable estimation results of robot localization and landmarks estimation. Based on these result, it is indeed shows consistency with the results obtained in [3] with a slight improvement from H_∞ filter. Belong to this results of fast convergence, process time for SLAM may reduce significantly and definitely nurture the SLAM problem. These results inspire further achievement and development of H_∞ filter.

6. CONCLUSIONS

H_∞ filter is still new to SLAM and need further analysis and development to achieve better motivating results. Moreover, H_∞ filter is capable to approximate linear and non-linear system that has wide coverage and variety of

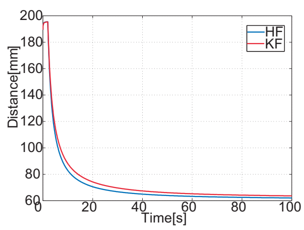

Fig. 11 Estimation of landmark 2

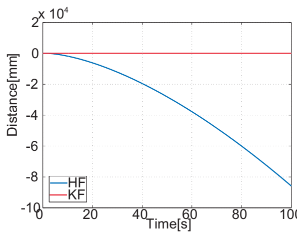

Fig. 12 The landmarks estimation:Effect of $R >> \gamma$

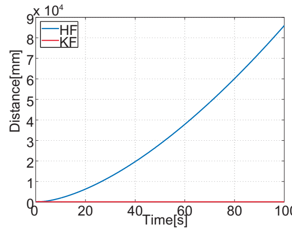

Fig. 13 MSE for the landmarks estimation:Effect of $R >> \gamma$

noise which are useful for SLAM problem. These results thus consistent with the fundamental lies in H_∞ filter where the designer should consider appropriate level of weighting noise of Q, and R to achieve certain level of performance.

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