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The Effect of Breakup of Melting Snowflakes on the Resulting Size Distribution of Raindrops

By **Yasushi Fujiyoshi**

Institute for Hydrospheric-Atmospheric Sciences, Nagoya Univ., Nagoya 464-01, JAPAN

and

Kenichiro Muramoto

Department of Electrical and Computer Engineering,
Faculty of Technology, Kanazawa Univ., Kanazawa 920, JAPAN

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Abstract

The effect of breakup of melting snowflakes on the resulting size distribution of raindrops was discussed based on the breakup behavior of snowflakes as they melted in warm kerosene. The maximum diameter, cross-sectional area, and mass of 50 snowflakes were measured as well as the size distribution of the water drops resulting from their melting. The total number of resulting water drops correlated best with the original mass of the snowflake. The averaged number of water drops increased linearly with an increase in mass for masses less than 3.0 mg. Although the mass of each snowflake was similar, the size distribution of the resulting water drops varied greatly. On average, the size of the water drops formed from snowflakes with a mass less than 1.0 mg and greater than 2.0 mg showed exponential and Gaussian distributions in their percentage of original snowflake mass, respectively. Taking into account only the breakup of melting snowflakes, we calculated the size distribution of raindrops formed from snowflakes having a Gunn-Marshall distribution. The slope of the calculated size distribution of raindrops agrees well with that of Marshall-Palmer distribution.

1. Introduction

In the famous paper of Gunn and Marshall (1958), they pointed out that there should be considerable breakup of the larger particles when snow turns to rain at the melting level. So far, many radar studies of the "bright band" have been performed, including those by Ryde (1946), Austin and Bemis (1950), du Toit (1967), Takeda and Fujiyoshi (1978), Yokoyama *et al.* (1984), Klaassen (1988), Russchenberg and Ligthart (1993), Fabry *et al.* (1994), and Hardaker *et al.* (1995). Ground observations have been made by Magono and Arai (1954), Ohtake (1969), and Yokoyama *et al.* (1985). In situ measurements using research aircraft have been made by Stewart *et al.* (1984) and Willis and Heymsfield (1989). Stewart *et al.* (1984) stated that the radar and in situ observations can largely be explained without considering appreciable ice particle breakup within the melting layer. Klaassen (1988) numerically simulated the melting layer and indicated that spontaneous breakup of snowflakes near the end of the melting improved the simulation. However, it remains unclear whether the snowflakes break up into fragments at this layer, or not.

Although there are some theoretical and experimental studies of melting ice spheres (Mason, 1956; Drake and Mason, 1966; Rasmussen and Pruppacher, 1982; Rasmussen *et al.*, 1984), there are few experimental studies of melting snowflakes. Matsuo and Sasyo (1981) held snowflakes on a net of thin nylon threads and melted them from beneath with warm air. Mitra *et al.* (1990) melted snowflakes in a more realistic environment using a vertical wind tunnel. Their experimental results agreed well with a theoretical heat transfer model which they presented. They also reported that spontaneous breakup was not generally observed, but, in snowflakes with a strongly asymmetric mass dis-

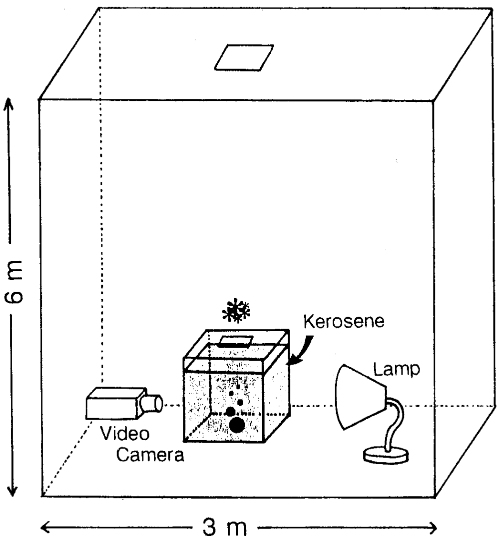

Fig. 1. The arrangement of the instruments. Snowflakes fell through a hole in the roof of the tower and into a kerosene-filled acrylic box.

tribution, small water drops were found to tear off from the rim of the flake, as was hypothesized by Knight (1979) and Fujiyoshi (1986). A quantitative measurement of the number of drops formed from a single melting snowflake would enable us to estimate the change in size distribution of precipitation particles in the melting layer.

The purpose of this study is to discuss the possible effect of breakup of melting snowflakes on the resulting size distribution of raindrops based on some experimental results. Assuming some adequate assumption of the breakup process, we can simulate the change of size distribution from Gunn-Marshall's to Marshall-Palmer's. However, this kind of study would be just theoretical. So far, various approximations about the distribution of water within the melting snowflake have been made to simulate the "bright band" (*e.g.* ice core with water shell (Aden and Kerker, 1951); ice-water-air mix (Klaassen, 1988); ice-air mixture with a liquid-water shell (Hardakeretal., 1995)). We know that these are not realistic simulations of melting snowflakes and there is a need to discuss this problem based on some quantitative experiments.

For this purpose, the breakup behavior of snowflakes as they melted in warm kerosene was studied. We admit that the melting behavior of snowflakes in kerosene is quite different from that in the air. However, the experimental system had the advantage that all water drops made from one snowflake could be counted. In a vertical wind tunnel it is quite difficult to sustain and count all water drops from a snowflake, because the various sizes of drops have a wide range of terminal velocities. Using our experimental system, we were able to discuss quantitatively the correlations between the size distribution of water drops per snowflake and the mass, size, cross-sectional area of a snowflake for the first time. The high viscosity and heat capacity of the kerosene caused the snowflakes to disintegrate into a larger number of water drops than would occur

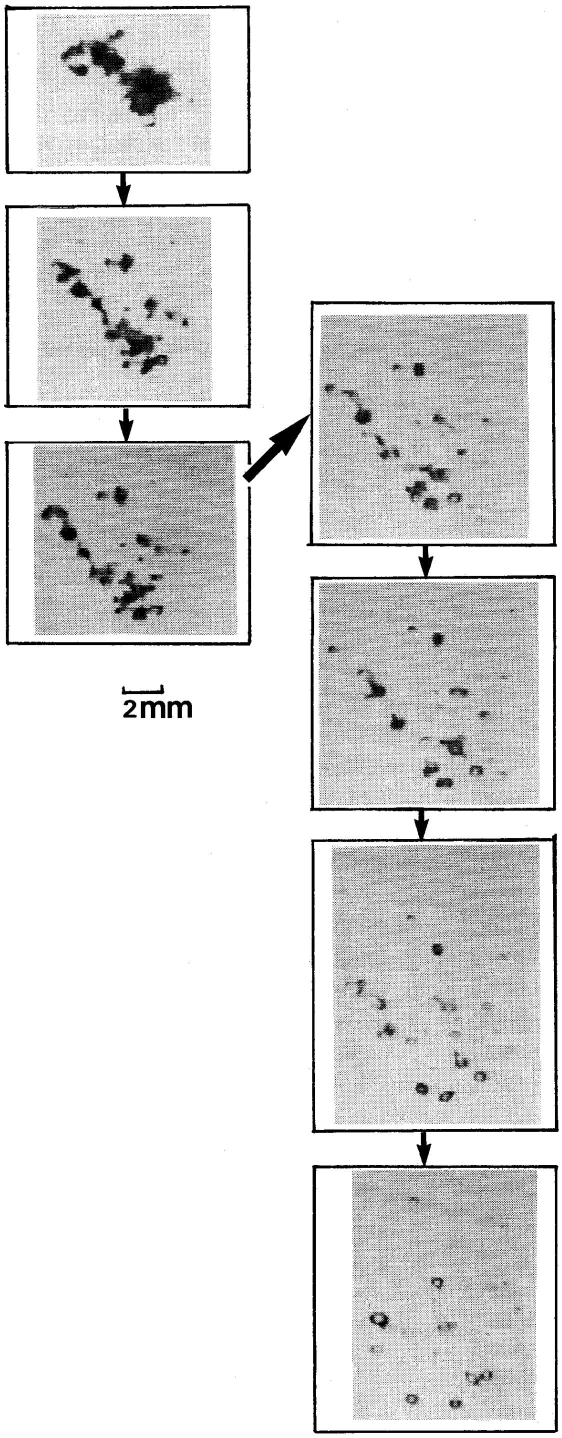

Fig. 2. Sample video images of the melting process of a snowflake. Snowflakes melted completely in several seconds.

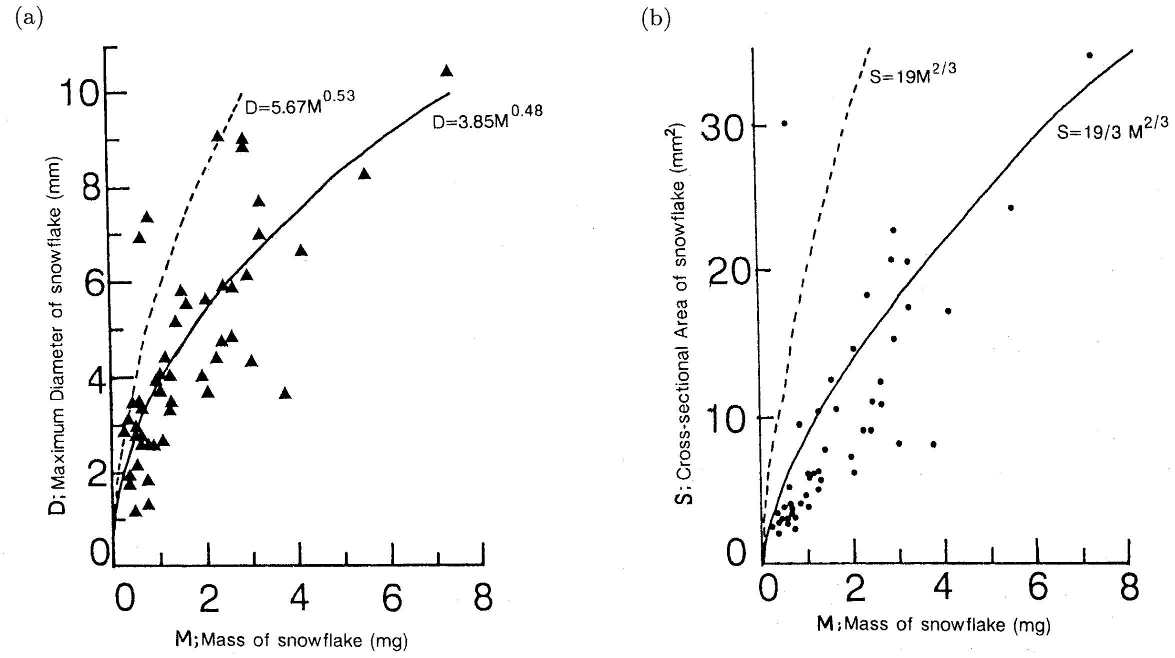

Fig. 3. (a) Maximum diameter (D) versus mass (M) of snowflakes. Also depicted are empirical relationships for graupel-like, lump type snow (solid line: Locatelli and Hobbs (1974)) and for snowflakes composed of heavily rimed dendritic crystals (dashed line: Locatelli and Hobbs (1974); Mitchell *et al.* (1990)). (b) Cross-sectional area (S) versus mass (M) of snowflakes. The dashed line represents the empirical relationship obtained by Sasyo and Matsuo (1980). The solid line is calculated on the assumption that the values of S are equal to one-third of those of the cross-sectional areas measured by Sasyo and Matsuo (1980).

during melting in air. Therefore, the discussion presented here would be an extreme effect of breakup. The opposite extreme is the experimental result reported by Mitra *et al.* (1990).

2. Instrument and data analysis

Figure 1 shows the experimental system, which was located on the roof of Kanazawa Univ. at Kanazawa City, Japan, near the Sea of Japan. Snowflakes fell through a hole in the roof of the tower and into a 4×4 cm hole in the top of the transparent acrylic box. The box with 30×30 cm cross section and 50 cm in depth was filled with kerosene. Snowflakes melted completely in several seconds. A CCD video camera recorded the melting of the snowflakes, which were backlit by a lamp. The field of view of the camera was 38×41 mm.

Figure 2 illustrates the melting process of a snowflake. The video output was digitized by an image processor into a region of resolution 480×512 dots, each with 8 bits of gray-scale information. The size of each drop could be determined within 0.08 mm. The following characteristic values were calculated for each snowflake: the maximum diameter (D) and the cross-sectional area (S). It is to be noted that the measured cross-sectional area is the vertical cross-sectional area, and the measured maximum diameter is not necessarily equal to the actual maximum diameter, since the video camera took pictures of them from only one side. The mass of a snowflake (M) was calculated as the total mass of the resultant water drops.

3. Results of measurements

Measurements were made on February 2, 1993. The temperature of the kerosene in the tank was about 9°C. Though the air temperature was not measured, it was just higher than 0°C. Past studies suggest that the breakup of snowflakes predominantly occur in their later melting stage. Therefore, the snowflakes observed would correspond to those just prior to breakup. A total of fifty snowflakes was measured. Their maximum diameters ranged from 1.19 to 10.43 mm, their mass, from 0.26 to 7.33 mg, and their cross-sectional areas, from 2.0 to 34.5 mm^2. Most of the observed snowflakes were composed of heavily rimed dendritic crystals.

Figures 3a and 3b show the relationships between M and D, and M and S, respectively. Both D and S correlated well with M. The $M - D$ relationship is described well by the empirical relationship reported by Locatelli and Hobbs (1974) obtained for the lump-type of graupel-like snow (solid line, Fig. 3a). While the snowflakes observed were composed of heavily rimed dendritic crystals, the $M - D$ relationship of snowflakes of this type pre-

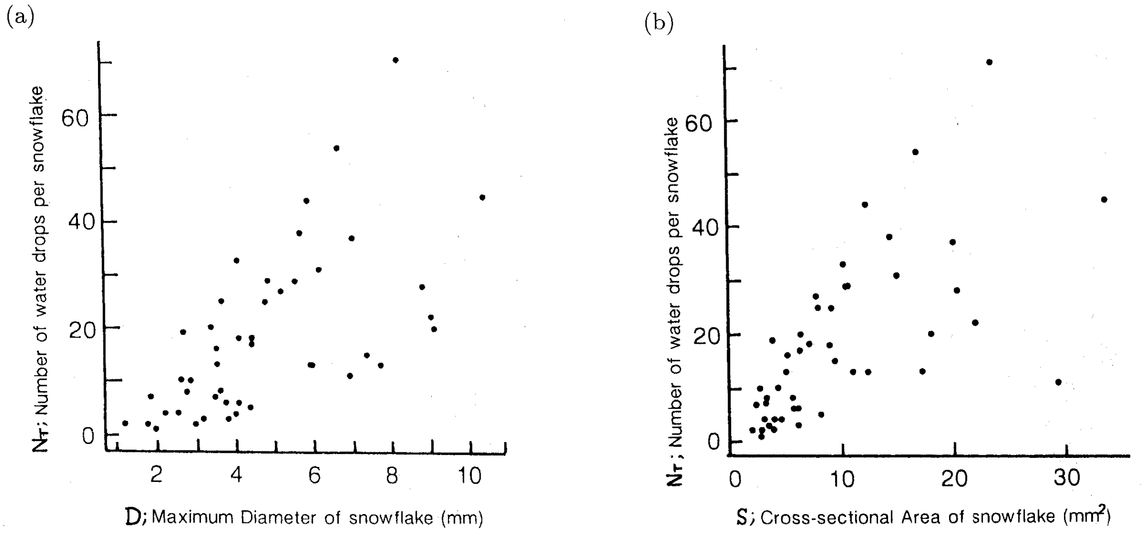

Fig. 4. Number of water drops per snowflake (N_T) versus (a) maximum diameter (D); (b) cross-sectional area (S); (c) mass (M) of snowflakes. The open circle shows the average number (N_{ave}) of water drops disintegrated from all snowflakes within each 0.5-mg increment of the total mass (M), respectively. The dashed line is a regression line expressed by Eq. (2).

sented by Locatelli and Hobbs (1974), and Mitchell *et al.* (1990) (dotted line, Fig. 3a) did not describe our data. The discrepancy may be explained by the fact that graupel-like particles were often included in the snowflakes in our experiment and that the snowflakes were slightly wet, since the air temperature was nearly 0°C on the ground.

The dashed line shown in Fig. 3b is the $M - S$ (M: mg, S: mm^2) relationship reported by Sasyo and Matsuo (1980), that is,

$$S = 19M^{\frac{2}{3}}. \tag{1}$$

Our data differ markedly from their measurements. This would be mainly because Sasyo and Matsuo (1980) measured the horizontal cross-sectional areas of snowflakes, while we measured the vertical cross-sectional areas. A better correlation to the experimental data is obtained with the additional assumption that the ratio of the horizontal to vertical cross-sectional areas is one-third (solid line, Fig. 3b), that is,

$$S(\text{vertical}) = S(\text{horizontal})/3.$$

Figures 4a, 4b and 4c show the relationships between the total number of water drops (N_T) and D, N_T and S, N_T and M, respectively. These figures show that both the maximum value and the variation of N_T increase with increasing D, S, and M. Snow- flakes that broke into a large number of water drops had large D, S, and M, but those with large D, S, and M did not necessarily break into a large number of water drops. This reason will be discussed in section 5.1. The correlation between N_T and M is the strongest. The correlation coefficients are 0.66 ($N_T - D$), 0.68 ($N_T - S$), and 0.77 ($N_T - M$).

Figure 4c implies that N_T increases with the mass of snowflakes. This tendency is quantitatively presented by a dashed line in Fig. 4c. In the figure, the average number of water drops (N_{ave}) was calculated per snowflake grouped into 0.5-mg mass increments of M. N_{ave} increases linearly with increasing mass for mass less than 3.0 mg. The $N_{\text{ave}} - M$ (mg) relationship is approximated by the equation:

$$N_{\text{ave}} = 1 + 11M. \tag{2}$$

It is expected that the types and sizes of snow particles comprising snowflakes did not change substantially with time. Thus masses would be proportional to the number of snow particles. Physically, breakup of a snowflake means the separation between snow particles. The separation occurs at the branches connecting them where melting proceeds quickly. The number of branches would be in proportion to the number of snow particles. Therefore, the result shown above indicates that the total number of raindrops would be approximately in proportion to the total number of branches which a snowflake contains.

4. Detailed analysis of the $N_T - M$ relationship

As mentioned above, N_T had the highest correlation to the mass of the original snowflake (Fig. 4c). To allow for a more detailed statistical analysis, we classified the snowflakes into three classes according to their masses: Class 1 ($0 - 1.0$ mg), Class 2 ($1.0 - 2.0$ mg), and Class 3 ($2.0 - 3.0$ mg). Class 1 had 19 snowflakes, Class 2 had 13 snowflakes, and Class 3 had 11 snowflakes. We omitted snowflakes with a mass larger than 3.0 mg in this analysis because they were few in number (total 7).

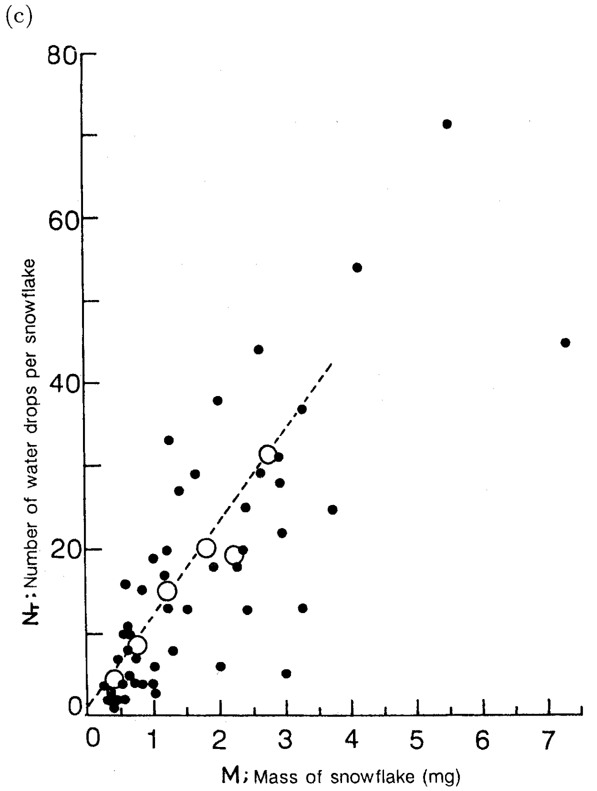

Fig. 4. (Continued)

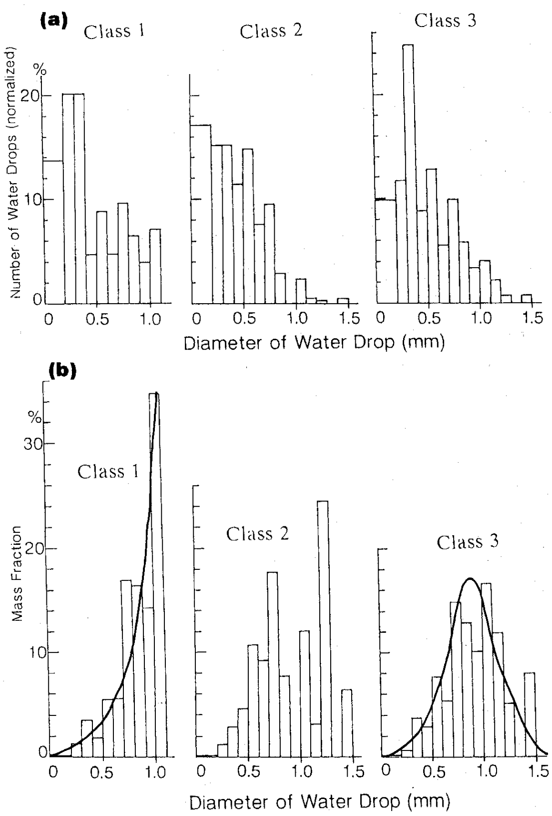

Fig. 5. (a) Size distributions of water drops in 0.1-mm increments of size which disintegrated from snowflakes in Classes 1 (0 − 1.0 mg), 2 (1.0 − 2.0 mg), and 3 (2.0−3.0 mg).Each distribution was normalized by dividing by the total number of water drops. (b) Frequency distributions of the mass of water drops in 0.1-mm increments of size for snowflakes in Classes 1, 2, and 3. The distributions of Class 1 andClass 3 are approximately exponential and Gaussian (solid line), respectively.

Figure 5a shows the size distributions of the water drops disintegrated from each class of snowflakes. The drops were separated into groups based on drop diameter; there was a 0.1-mm difference in diameter between the groups. Each distribution was normalized by dividing by the total number of water drops. The size of the water drops ranged widely for all classes of snowflakes. In Class 1, most of the water drops had a small diameter ($\leq$ 0.4 mm). However, the normalized number of large water drops in Class 1 is greater than that in Class 2 or Class 3. The number of drops of each size decreases more rapidly with increasing size in Class 3 than in Class 2.

Figure 5b shows the mass distributions of the water drops. The drops were grouped by diameter, as above, and the mass of each group was totalled. The mass was normalized by dividing by the total mass of the water drops in each class (*i.e.*, the total mass of the snowflakes). In Class 1, most of the mass of the melted snowflakes was distributed to large water drops ($\geq$ 0.7 mm), with the distribution to the largest drops being especially high. That is, a relatively greater percentage did not break up. The distribution was approximately exponential (solid line). On the other hand, when the mass of the snowflake was larger than 3.0 mg, nearly all broke into many water drops. In Class 3, the distribution was approximately Gaussian (solid line; $\sigma = 0.15$). In Class 2, the distribution shows a mixture of the trends in Class 1 and Class 3 (see Appendix A for the details of the distribution functions).

5. Discussion

5.1 Mass distribution in a snowflake

Figure 6 shows the average number and the standard deviation of water drops per snowflake grouped into 1.0-mg increments of M and 1.0-mm increments of D. The values were not shown when the number of snowflakes was less than three in each domain. As expected, the average number of water drops increases with an increase in both D and M. In all cases the standard deviation is close to half of the average number. A difference in the degree of asymmetric mass distribution in a snowflake would be one of the main factors which could cause such large standard deviation, as suggested by Fujiyoshi (1986) and Mitra *et al.* (1990). As it is difficult to measure the mass distribution of individual snowflakes, we discussed the relationship between the degrees of breakup and the mass distribution in a snowflake by using the normalized moment (I) as follows.

We assumed that the darkness is proportional to the density of mass at that point, and calculated the normalized moment (I). The definition of the normalized moment is presented in Appendix B. The small (or large) value of the normalized moment means the high (or low) concentration of mass in a snowflake. Therefore, a large value of the normalized moment corresponds with the high degree of asymmetry in the mass distribution in a snowflake. No correlation was detected between I and D, I and M, and I and S (not shown). The frequency distribution of I of all snowflakes (Fig. 7) shows a clear Gaussian distribution.

Although the moment of snowflakes does not correlate with N_T (not shown), it still affects the degree of breakup. To depict this, we compared the average moment of "fragile" snowflakes with that of "hard" ones. "Fragile" snowflakes are defined as those whose ratio of the measured number of water drops to the calculated number (using Eq. 2) is larger than 1.5, and "hard" snowflakes are those with ratios smaller than 0.5. The average value ($\pm$standard deviation) of the moments of all "fragile" snowflakes was 0.49 ± 0.11, and that of "hard" snowflakes was 0.37 ± 0.06. The "fragile" snowflakes occupy the upper portion of the distribution of I, as shown in Fig. 7, while the "hard" snowflakes occupy the lower portion.

The result shown in Fig. 7 would explain the reason why the mass of resultant water drops disintegrated from large snowflakes showed the Gaussian distribution in Fig. 5b. As a snowflake melts, the portions where snow particles densely aggregate within it become clusters (Fujiyoshi, 1986). Since the clusters become raindrops, the size distribution of raindrops is closely related to the mass distribution of clusters, hence to the degree of mass concentration in a snowflake. Figure 7 shows that the degree of asymmetry of mass distribution changes almost randomly from snowflake to snowflake, regardless of D, M, and S. This result strongly suggests that the averaged mass distribution of clusters of total snowflakes within some mass range would also show a Gaussian distribution. If the distance, however, between the clusters is small, they would coalesce with each other due to surface tension of water. The distance between the clusters of small snowflakes would generally be smaller than that of larger snowflakes. This would be the reason why a relatively greater percentage of small snowflakes (Class 1) did not break up.

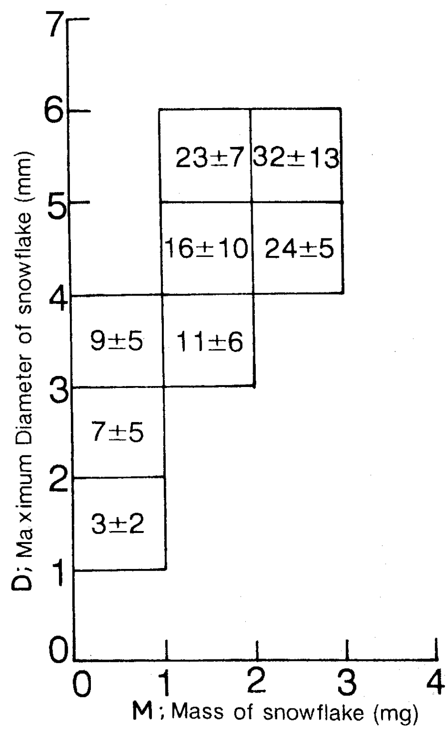

Fig. 6. Average number (N_{ave}) and the standard deviation of water drops disintegrated from all snowflakes within each 1.0-mg increment of the total mass (M) and each 1.0-mm increment of the maximum diameter (D).

5.2 Size distribution of raindrops resulting from the melting of snowflakes

As the microphysical processes are quite complicated in the melting layer, no complete simulation of the melting layer has so far succeeded. Here, we discuss only the effect of breakup on the change in size distribution when snowflakes melt into raindrops. We calculated the size distribution of raindrops assuming that no coalescence occurred and melting snowflakes broke up in the same manner as presented in Fig. 5b (see the detailed explanation of the calculation in Appendix A).

Figure 8 shows the size distribution of snowflakes (Gunn and Marshall) and raindrops (Marshall and Palmer, and the present study). The calculated size distribution (thick line) is steeper than that of G&M (dotted line), and shows a good agreement with M&P (thin line) for a precipitation intensity of 0.5 mm hr^{-1}. Although the discrepancy in number density between the calculated and M&P distributions increases with an increase in precipitation intensity, the slope of the calculated size distribution is almost parallel with that of M&P. The discrepancy becomes large with an increase in precipitation intensity. The reason is as follows:

When we calculate the precipitation intensity by using Eq. (A-1)(or (A-3)) and (A-2)(or (A-9)), the

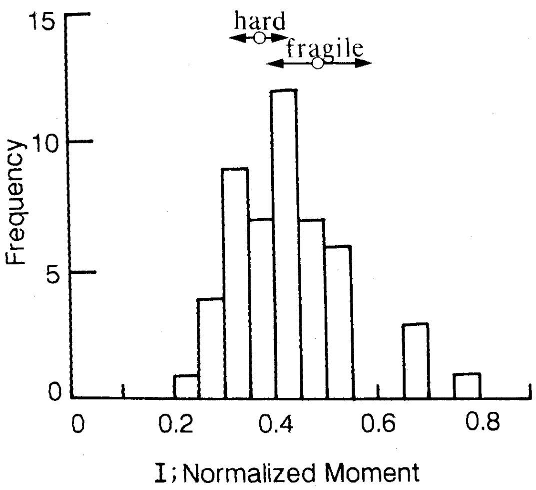

Fig. 7. Frequency distribution of normalized moments of all snowflakes. See the text about the definition of "hard" and "fragile" snowflakes.

intensity is larger than the precipitation intensity which is given as a parameter to Eq. (A-1)(or A-3)). For example, when we give $R = 0.5$, 2.0 and 4.0 mm hr^{-1} as values of the parameter, the calculated intensities are $R = 0.6$, 1.3 and 4.7 mm hr^{-1} for raindrops, and $R = 0.7$, 3.3 and 7.4 mm hr^{-1} for snowflakes. This is caused by the inconsistency between Eq. (A-1)(or (A-3)) and (A-2)(or (A-9)), as pointed out by Zawadzki and Antonio (1987) and Zawadzki *et al.* (1994). When we used the velocity-mass relationship of the conical graupel ($V_s = 2.5M^{0.28}$; Locatelli and Hobbs (1974)) instead of the relationship of the lump-type of graupel-like snow ($V_s = 1.4M^{0.08}$; Locatelli and Hobbs (1974)), the calculated number density increased about three times and $R = 8.4$ mm hr^{-1}. On the other hand, a smaller fall speed ($V_s = 0.7M^{0.08}$) resulted in decreasing number density and $R (= 1.7$ mm hr$^{-1})$. Therefore, the change in terminal velocity of snowflakes causes much difference in calculated precipitation intensity and number density of raindrops, but not in the slope. Theoretically, we must make these size distributions using snow/water content (g m^{-3}) as a parameter.

The calculated distributions show large increasing rate in number density for the small raindrops (≤ 0.5 mm) [1]. The M&P distribution gives that of raindrops arriving at the ground. Therefore, evaporation of raindrops would cause a decrease in the number of small raindrops, in addition to that of the coalescence process which was ignored in this study.

6. Summary

We recorded the breakup behavior of snowflakes in warm kerosene using a video camera and analyzed the resultant images using an image processor. We then measured the size distribution of water drops formed from 50 snowflakes, as well as each snowflake's maximum diameter, cross-sectional area, and mass. The mass of snowflakes correlated well with both D and S. The total number of resultant water drops showed the best correlation with mass; the average number of water drops increased linearly with increasing original snowflake mass for mass less than 3 mg. Although the mass of the snowflakes were very similar overall, the size distribution of water drops varied widely. The condition that a snowflake has large M, D, and S is necessary, but not sufficient, to result in its breakup into a large number of water drops. The moment of a snowflake affects the degree of breakup, and is a measure of the degree of asymmetry in the arrangement of mass in a snowflake. The average moment of "fragile" snowflakes was larger than that of "hard" snowflakes, indicating its influence on the degree of breakup.

When the snowflake mass was less than 1.0 mg, a relatively greater percentage did not break up. On the other hand, when the mass of the snowflake was larger than 3.0 mg, nearly all broke into many water drops. The size of the water drops formed from snowflakes with a mass less than 1.0 mg and greater than 2.0 mg showed exponential and Gaussian distributions in their percentage of original snowflake mass, respectively.

Based on these experimental results, we discussed the possible role of the breakup of melting snowflakes on resulting size distribution of raindrops. The slope of the calculated size distribution showed a good agreement with that of M&P distribution, although there was a discrepancy in number density between them.

7. Conclusion

Only taking into consideration the spontaneous disintegration of water drops, Komabayashi (1965) suggested that M&P formula represents a stationary state of size distribution of raindrops. Young (1975) also attributed the M&P distribution to the processes that do not involve the solid phase of precipitation. Recently, however, Zwadzki *et al.* (1994) concluded that Young's (1975) explanation for the M&P distribution is incorrect, and stated that the M&P distribution results from processes affecting particles in the solid phase rather than from the interaction of raindrops.

This study is based on the assumption that raindrops with the M&P size distribution fall on the ground when snowflakes with the G&M size distribution fall above the melting level. Our study

1 Dr. Y. Asuma (1995, private communication) found that a large number of small water droplets in the melting layer.

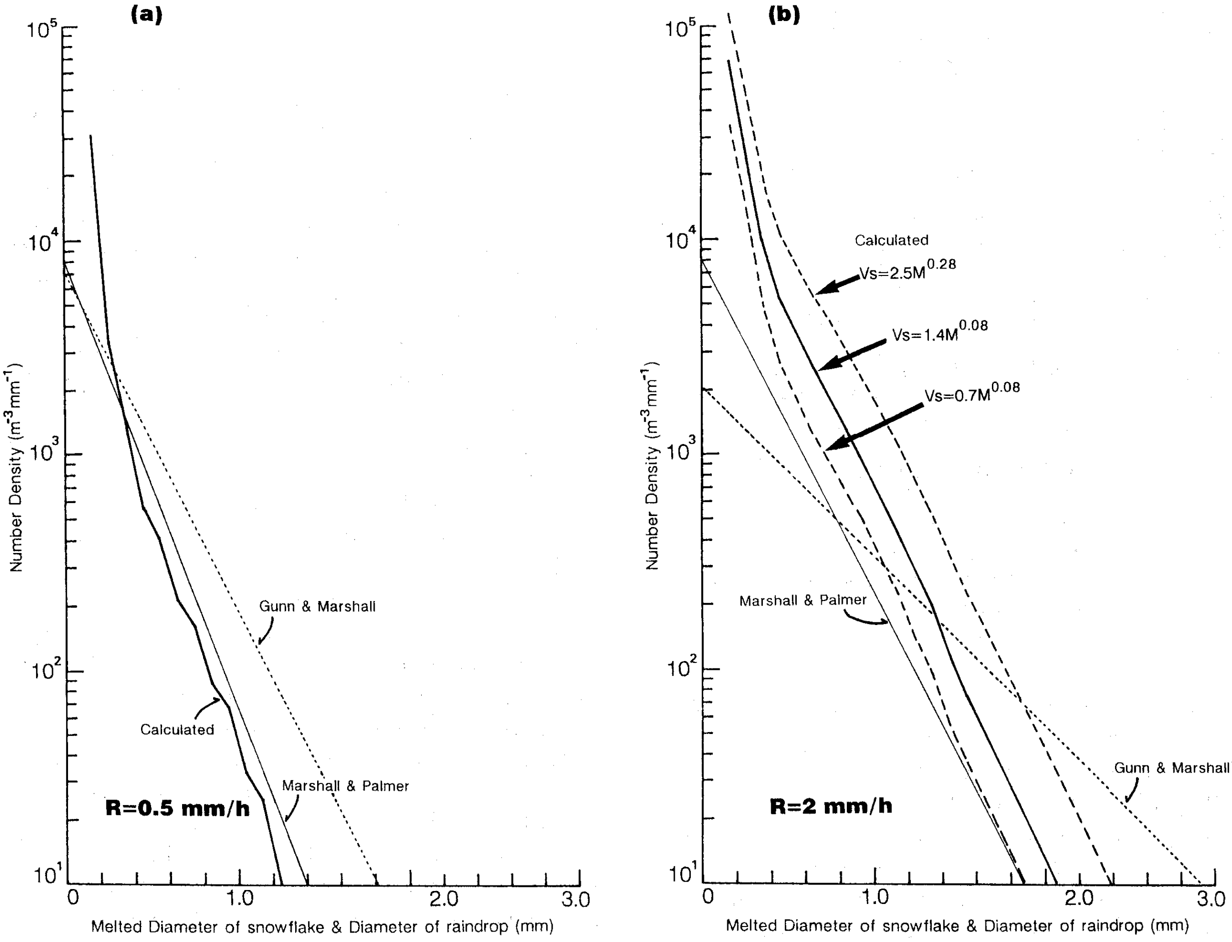

Fig. 8. The size distributions of snowflakes (Gunn and Marshall; dotted line), raindrops (Marshall and Palmer; solid line), and calculated raindrops (present study; thick solid line). (a) $R = 0.5$ mm hr^{-1}; (b) $R = 2.0$ mm hr^{-1}; (c) $R = 4.0$ mm hr^{-1}

showed that the breakup of melting snowflakes could account for the change in size distribution from G&M's to M&P's. It is inferred that aggregation of snowflakes proceeds in the melting layer. If the giant snowflakes are formed in the melting layer and their contribution to the mass distribution of snowflakes is large, the number density of large raindrops would be larger than that expected from the snowflakes with a G&M distribution. That is, the resultant size distribution of raindrops would differ from M&P's. Therefore, if the basic assumption holds, most rain drops would originate from the snowflakes whose sizes/masses are within the size range of snowflakes before melting, even though snowflakes aggregate and increase their sizes/masses in the melting layer. In other words, giant snowflakes hardly contribute to the size distribution of raindrops, though they mainly contribute to the radar echo intensity in the melting layer.

We admit the critique that the relevance of the present result to melting snowflakes in free-fall in the atmosphere is doubtful. However, we would like to emphasize that the present study firstly provided a quantitative discussion about the effect of breakup on the change in the size distribution of melting snowflakes. All of optical spectrometers instrumented on aircraft can measure D and S of snowflakes. However, we need some device which enables in situ measurement of the mass of snowflakes, since the total number of resultant water drops showed the best correlation with M of snowflakes.

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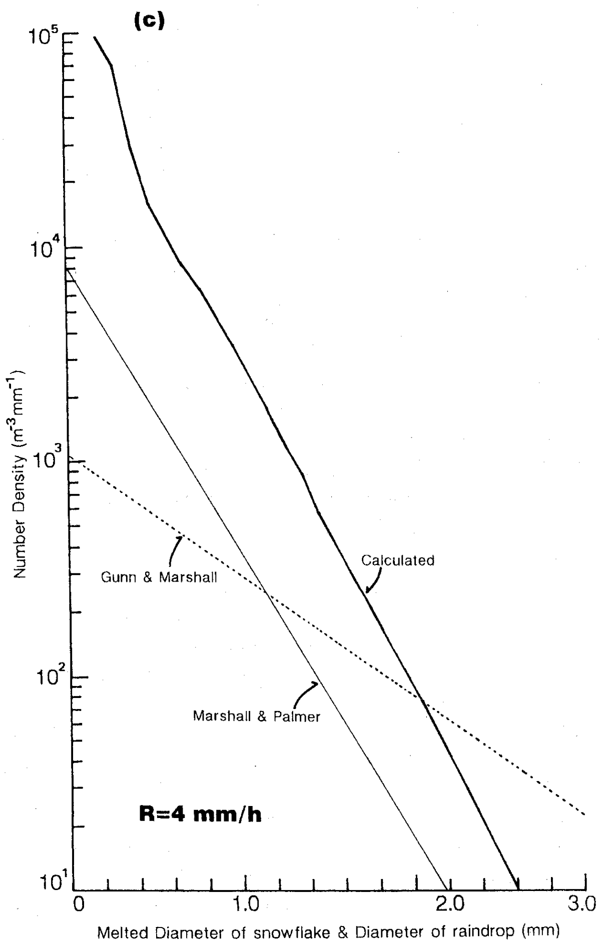

Fig. 8. (Continued)

Appendix A

The size distribution of snowflakes is given by Gunn and Marshall (1958):

$$N_s(D) = D_{s_0} e^{-\Lambda_s D}$$
$$N_{s_0} = 3.8 \times 10^3 R^{-0.87} m^3 mm^{-1},$$
$$\Lambda_s = 25.5 R^{-0.48} \qquad \text{(A-1)}$$

As D is the melted diameter of the snowflake, the mass of the snowflake is easily calculated ($M = \pi D^3/6$). As the observed mass-size relationship shown in Fig. 3b was well approximated by the relationship obtained for the lump-type of graupel-like snow (Locatelli and Hobbs, 1974), we used the velocity-mass relationship of this type of snow:

$$V_s(M) = 1.4 M^{0.08} \qquad \text{(A-2)}$$

Here, mass M is given in milligrams, diameter D in millimeters, and fall speed V_s in meters per second.

The size distribution of raindrops is given by Marshall and Palmer (1948):

$$N_r(\phi) = N_{r_0} e^{-\Lambda_r \phi}$$
$$N_{r_0} = 8.0 \times 10^3 m^{-3} mm^{-1}, \Lambda_r = 41 R^{-0.21} \qquad \text{(A-3)}$$

The mass frequency distribution of water drops formed from the snowflakes of Class 1 ($M \leq 1.0$ mg) in Fig. 5b is approximated by the exponential function:

$$F(\phi_n) = A\{e^{3.71\phi_n} - 1\} \qquad \text{(A-4)}$$

where ϕ_n is a normalized diameter $\phi_n (= \phi/D)$. The mass frequency distribution of water drops formed from the snowflakes of class 3 ($M \geq 3.0$ mg) in Fig. 5b is approximated by the Gaussian distribution function:

$$F(\phi_n) = B e^{-\frac{(\phi_n - 0.5)^2}{2\sigma^2}}, \sigma = 0.15 \qquad \text{(A-5)}$$

The coefficients A (= 10.26) and B (= 266) are determined so that,

$$\int_0^1 F(\phi_n) d\phi_n = 100 \qquad \text{(A-6)}$$

Since the mass distribution of the water drops formed from the snowflakes in Class 2 ($1.0 \leq M \leq 2.0$ mg) showed a mixture of the trends in Class 1 and Class 3 (Fig. 5b), we used Eq. (A-4) when $M \leq 1.5$ mg, and Eq. (A-5) when $M \geq 1.5$ mg.

The number of raindrops whose diameters fall between ϕ and $\phi + \delta\phi$ formed from a snowflake with its mass M is given by the following equation:

$$n(\phi, M) = M \left(\frac{F\delta\phi}{100}\right) \Big/ \left(\frac{\pi\phi^3}{6}\right) \qquad \text{(A-7)}$$

From the requirement of mass conservation, the number density of raindrops is given by:

$$n_d(\phi, M) = \frac{n(\phi, M) V_s(M)}{V_r(\phi)} \qquad \text{(A-8)}$$

Here, V_r is the fall velocity of raindrops, and we used the velocity-diameter relationship of raindrops given by Atlas *et al.* (1973):

$$V_r(\phi) = 9.65 - 10.3 e^{-0.6\phi} \qquad \text{(A-9)}$$

Total number density of raindrops between ϕ and $\phi + \delta\phi$ formed from total snowflakes with their melted diameters fall between D and $D + \delta D$ is given by:

$$n_r(\phi, D) = N_s(D) n_d(\phi, D) \frac{\delta D}{\delta\phi} \qquad \text{(A-10)}$$

Finally, total accumulated number density of raindrops formed from total snowflakes with their melted diameters from $D_{\min}$ to $D_{\max}$ ($D_{\min}$ = 0.2 mm and $D_{\max}$ = 4 mm in our calculation) is given by:

$$N_r(\phi)_{\rm cal} d\phi = \int_{D_{\min}}^{D_{\max}} n_r(\phi, D) dD \qquad \text{(A-11)}$$

Appendix B

The normalized moment of a snowflake is defined by the following equation:

$$I = \frac{\sum_k f_k d_k^2}{(\sum_k f_k)^2}, \tag{B-1}$$

where k is the position of a given sampling point on a snowflake and f_k is the darkness of the image (8 bits) at that point. The darkness of the image is normalized by dividing by the value of the darkest point (details given in Gonzarez and Woods, 1992). The value of d_k means the distance from the center of image gravity (X, Y) of a snowflake defined as

$$d_k^2 = (x_k - X)(y_k - Y) \tag{B-2}$$

Here, X and Y are defined as

$$X = \frac{\sum_k x_k f_k}{\sum_k f_k}, \; Y = \frac{\sum_k y_k f_k}{\sum_k f_k} \tag{B-3}$$

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融解雪片の分裂が雨滴の粒径分布に及ぼす効果

藤吉康志
（名古屋大学大気水圏科学研究所）
村本健一郎
（金沢大学工学部電気・情報工学科）

暖めたケロシン中で雪片を融解させ、その際形成される水滴の粒径分布を調べた。この実験結果を基に、雪片の融解分裂過程が、雨滴の粒径分布に与える効果について考察した。合計50個の融解前の雪片について、その最大直径、断面積、質量も同時に測定した。1つの雪片から生じる水滴の総数は、雪片の質量と最も相関が高く、少なくとも質量が3.0 mg 以下の場合には、平均的な水滴の個数は質量と共に直線的に増加した。ただし、質量が同じでも形成される水滴の粒径分布には大きなバラツキがあった。生成された水滴の平均粒径分布は、質量が1.0 mg 以下の場合は指数関数で、2.0 mg 以上の場合はガウス分布で近似出来た。初め Gunn-Marshall 型の粒径分布をしていた雪片が、ここで得られた実験式にのって融解分裂したと仮定すると、得られた雨滴の粒径分布の勾配は Marshall-Palmer 分布の勾配と極めて良く一致した。