H_∞ Filter-SLAM: A Sufficient Condition for Estimation ⋆

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Abstract: A theoretical study about H_∞ Filter is proposed to determine a sufficient condition for estimation purposes. Two different cases of initial state covariance are analyzed to guarantee that a best solution for SLAM problem is achieved with consideration about process and measurement noises. If the conditions are not satisfied, then the estimation exhibit unbounded uncertainties and consequently result in erroneous inference. Simulation result shows consistency as suggested by the theoretical analysis. These results consistently supports and guarantees our previous findings.

Keywords: Sufficient condition, Finite Escape Time, H_∞ Filter, SLAM, Estimation

1. INTRODUCTION

Uncertainties exist in different kinds of forms. Their presence inherently causing various kinds of applications to suffer. Apparently, this is unavoidable and even though some devices are proposed to favor the problem, the solution still demand further improvements. One of the robotics applications that suffers from uncertainties is Simultaneous Localization and Mapping(SLAM) problem. It states a condition where a robot is assign to observe an unknown environment and incrementally building a fair knowledge about its surroundings. The robot then attempts to localize itself on the constructed map recursively. The SLAM illustration is provided in Fig.1.

A considerable approach that is seems able to tolerate uncertainties in SLAM is probabilistic. Hence, up to date, bayesian approaches is preferable in comparison to behavior-based SLAM and mathematical-based SLAM(See S.Thrun et.al (2005)). Extended Kalman Filter(EKF)-SLAM as proposed by G.Dissayanake et.al (2001) has received a wide attention as it is easy to apply and has lower computation cost than any other probabilistic approaches. Unfortunately, SLAM demand further considerations about the environment conditions. An assumption of gaussian noise has restricting EKF role as the main player and thus offering places for more robust approach such as the Particle Filter(PF). However, PF has some drawbacks; computational cost, complex. Therefore, in this sense, we propose H_∞ Filter approach for SLAM as it more robust than EKF and has lower computational cost than PF. A brief description of the filter is introduced by D.Simon (2001).

In this paper, we analyze further H_∞ Filter based SLAM performance to aid previous works by M.E.West et.al (2006); H.Ahmad et.al (2009, 2010). M.E.West et.al (2006) have proved that H_∞ Filter is a solution for SLAM problem. Its performance has been compared to PF and EKF for underwater application. Even though PF shows better results, H_∞ Filter still the best solution in terms of computational cost and in non-gaussian noise environments as claimed by H.Ahmad et.al (2009, 2010). Unlike EKF, as reported by H.Ahmad et.al (2009); P.Bolzern et.al (1997), H_∞ Filter solution can unboundedly increased and exhibit *Finite Escape Time*. Therefore, to apply H_∞ Filter efficiently in SLAM, designer must carefully design its parameters to achieved a desired performance. P.Bolzern et.al (1999) discovered that H_∞ Filter must also satisfy $P_0 = R^{-1}$ to achieve better estimation. They proposed a study regarding both filtering and prediction and found that under a feasibility and sufficient condition, the filter achieved a stable results. Besides, H.Ahmad et.al (2010) proposed the covariance inflation and γ-switching strategy as an additional tools to avoid *Finite Escape Time*. Experimental results validates their analysis and proved that those two method can prevents *Finite Escape Time*.

With regards to the preceding works, a further analysis of H_∞ Filter-SLAM are proposed. We guarantee that if some conditions are satisfied, then H_∞ Filter gives better estimation while at the same time refraining the appearance of *Finite Escape Time*$(F.E.T)$ in the estimation. The results also proves and consistent with previous results. There are also some trade-off between γ and the design parameters especially about the initial state covariance, process and measurement noises distributions. Nevertheless as there are many types of SLAM approaches, two conditions of different initial state covariance are examine to understand its effect to SLAM with consideration about the process and measurement noises distributions. The analysis are shown to provide a recognizable effect in different situations of environment conditions.

We organize this paper as follows. Section II describes the problem formulation about SLAM problem. Then followed by Section III which examines the convergency of H_∞ Filter for SLAM under some conditions. Next, we demonstrates some simulation results which consists of two cases to evaluate our proposal in Section IV. Finally, Section V concludes our paper.

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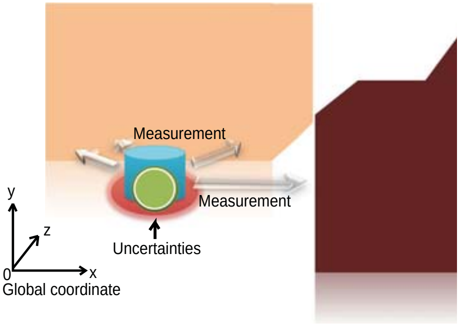

Fig. 1. Simultaneous Localization and Mapping: Robot taking relative measurements with some uncertainties

2. PROBLEM FORMULATION

H_∞ Filter problem describes that, for a given $\gamma > 0$, an H_∞ Filter attempts to find a solution for an estimated state $\hat{x}_k$, that satisfies

$$\sup_{x_0,v,w} \frac{\sum_{k=0}^{N} ||x_k - \hat{x}_k||}{\left\{ ||x_0 - \bar{x}_0||^2_{P_0^{-1}} + \sum_{k=0}^{N} ||v_k||^2_{R_k^{-1}} + \sum_{k=0}^{N} ||w_k||^2_{Q_k^{-1}} \right\}^{\frac{1}{2}}} < \gamma$$

where x_0, x_k is the robot($\in \mathbb{R}^3$) and landmarks($\in \mathbb{R}^{2m}, m = 1,2,\ldots,N$) states. w, v, are the process and measurement noises with covariance of Q_k, R_k respectively and $P_0 > 0$ is the initial state covariance. Furthermore, $Q_k \geq 0$, $R_k > 0$. The above equation alternatively means that the filter attempt to provide a solution when the estimation error to the noise ratio is less than a certain level of γ.

$$\theta_{k+1} = \theta_k + f_\theta(\omega_k, v_k, \delta\omega, \delta v) \quad (1)$$

$$x_{k+1} = x_k + (v_k + \delta v)T \cos[\theta_k] \quad (2)$$

$$y_{k+1} = y_k + (v_k + \delta v)T \sin[\theta_k] \quad (3)$$

$$L_{k+1} = L_k \quad (4)$$

where θ_k is the mobile robot pose angle, and ω_k, v_k are mobile robot turning rate and its velocity. $\delta\omega, \delta v$ are the associated noises for each ω_k, v_k respectively. While, x_k, y_k are the x, y cartesian coordinate of the mobile robot and $L_k \in \mathbb{R}^{2m}, m = 1,2,\ldots N$ is each respective landmark $x_i, y_i (i = 1,2,\ldots l)$ locations. T is the sampling rate. The process model for the landmarks is unchanged as the landmarks are assumed to be stationary and are given. In H_∞ Filter algorithm, the prediction state is given by

$$\hat{X}^-_{k+1} = f_k(\hat{X}_k, \omega_k, v_k, 0, 0) \quad (5)$$

where $\hat{X}_k \in \mathbb{R}^{(3+2m)\times(3+2m)}$ is the estimated augmented mobile robot and landmarks state with its associated covariance P_{k+1}.

$$P^-_{k+1} = \nabla f_r P_k [I - \gamma^{-2} P_k + \nabla H_i^T R_k^{-1} \nabla H_i P_k]^{-1} \nabla f_r^T + \nabla g_{\omega v} \Sigma_k \nabla g^T_{\omega v} \quad (6)$$

$\nabla f_r, \nabla g_{\omega v}$ are the Jacobian evaluated from the mobile robot

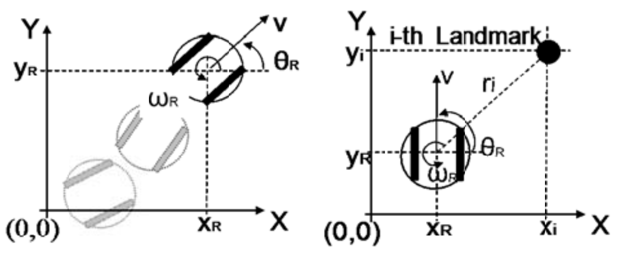

Fig. 2. Process(left) and Measurement Model(right) of SLAM

motion in (1)-(4) showing the Jacobian for robot and Jacobian for its associated noise respectively. Σ_k is the control noise covariance. For stationary landmarks and when $T = 1$,

$$\nabla f_r = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -v\sin\theta & 1 & 0 & 0 \\ v\cos\theta & 0 & 1 & 0 \\ 0 & 0 & 0 & I \end{bmatrix}, \nabla g_{\omega v} = \begin{bmatrix} \nabla f_{\omega v} \\ 0 \end{bmatrix} \quad (7)$$

where I is an identity matrix with an appropriate dimension. The mobile robot then makes measurements using its exteroceptive sensors and is shown by

$$z_i = \begin{bmatrix} r_i \\ \varphi_i \end{bmatrix} = \begin{bmatrix} \sqrt{(x_i - x_{k+1})^2 + (y_i - y_{k+1})^2} + v_i \\ \arctan \frac{y_i - y_{k+1}}{x_i - x_{k+1}} - \theta_{k+1} + v_{\theta_i} \end{bmatrix} \quad (8)$$

$$= H_i X_{k+1} + v_{r_i \theta_i} \quad (9)$$

where r_i and θ_i is the relative distance and measurements between robot and landmark i respectively. x_i, y_i show the respective landmarks number. This equation defines that the mobile robot measures relative distance and angle from a specific m^{th} landmark with some associated noises of v_i, v_{θ_i}.

Meanwhile, the mobile robot measurements about a landmark m is shown by using Jacobian as

$$\nabla H_i = \begin{bmatrix} 0 & -\frac{dx_k}{r} & -\frac{dy_k}{r} & \frac{dx_k}{r} & \frac{dy_k}{r} \\ -1 & \frac{dy_k}{r^2} & -\frac{dx_k}{r^2} & -\frac{dy_k}{r^2} & \frac{dx_k}{r^2} \end{bmatrix} = [-e - A_i A_i] \quad (10)$$

where $r = \sqrt{(x_i - x_k)^2 + (y_i - y_k)^2}$, $dx_k = x_i - x_k$ and $dy_k = y_i - y_k$.

$$e = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad A_i = \begin{bmatrix} \frac{dx_k}{r} & \frac{dy_k}{r} \\ -\frac{dy_k}{r^2} & \frac{dx_k}{r^2} \end{bmatrix}$$

The updated state covariance is given by below equations,

$$\psi = I_k + (\nabla H_i^T R_k^{-1} \nabla H_i - \gamma^{-2} I_k) P_k$$

$$P^+_{k+1} = \nabla f_r P_k \psi_k^{-1} \nabla f_r^T + \nabla g_{\omega v} \Sigma_k \nabla g^T_{\omega v} \quad (11)$$

where $K_k = P_k \nabla H_k^T (\nabla H_i P_k \nabla H_i{}^T + R)^{-1}$ with the corrected state update described by

$$\hat{X}^+_{k+1} = f_r \hat{X}_k + K_{k+1}(H_i X_k - H_i \hat{X}_k) \quad (12)$$

Notice that in (12) onward, H_m is replaced by H_i to indicate the Jacobian is evaluated at time k.

3. CONDITIONS FOR CONVERGENCE IN H_∞ FILTER-SLAM

The Fisher Information Matrix(FIM) is used to determine the updated state error covariance. If a robot starts moving from its initial position to point A and doing an observation at that point, FIM yield the following equation.

$$\Omega = \begin{bmatrix} P_{0v} & 0 \\ 0 & P_{0m} \end{bmatrix}^{-1} + \begin{bmatrix} -H_A^T \\ A^T \end{bmatrix}^T R_A^{-1} [-H_A \ A] - \gamma^{-2} I_n \quad (13)$$

$$= \begin{bmatrix} P_{0v}^{-1} + H_A^T R_A^{-1} H_A - \gamma^{-2} I_n & -H_A^T R_A^{-1} A \\ -A^T R_A^{-1} H_A & P_{0m}^{-1} + A^T R_A^{-1} A - \gamma^{-2} I_n \end{bmatrix} \quad (14)$$

where P_{0v}, P_{0_m} are the initial state covariance for robot and landmarks respectively. I_n is an identity matrix that hold an appropriate dimension. The landmarks are assumes to be stationary, so there are no noises affecting the prediction process for landmarks state. Equation (14) is said as the feasible condition(see P.Bolzern et.al (1999)) and is nontrivial to comprehend the behavior of the filter during estimation. After one time prediction, we have

$$P_k = P_{0v} + Q_k \tag{15}$$

Equation (15) explains that in every update, the state error covariance is also influenced by the process noise and play an important role to give a sufficient information in H_∞ Filter. The dissimilarity to Kalman Filter is belong to the existence of γ which attempts to reduce the system uncertainties in each observation. Equation (14) lead us to investigate below two cases of SLAM for different initial state covariance under consideration to process and measurement noises distributions.

(1) Robot initial state error covariance is smaller than the landmarks initial state covariance such that $P_{0v} << P_{0m}$
(2) Robot initial state error covariance is big and same to landmarks initial state covariance such that $P_{0v} = P_{0m}$

The first case define that the robot has more confidence about its location. This case relying on an assumption that robot has an efficient proprioceptive sensors. Next, the second case define that both robot and landmarks initial state covariance are unknown. This case is more likely the problem in real SLAM application as usually no prior information are available for reference. Provided by these two conditions, we suggest a theoretical study and analysis to comprehend their influences in SLAM problem. We investigate the effects of process and measurement noises to the estimation with a different cases of initial state covariance.

As been stated in references, the performance of H_∞ Filter is sensitive and depends on the design parameters such as the process and measurement noises and also the initial state covariance. We continue our study to describe explicitly that the selection of design parameters should satisfies some conditions to guarantee H_∞ Filter surpassing EKF performance(see H.Ahmad et.al (2010)). Furthermore, there are some particular trade-off which are necessary between the design parameters to achieve a best solution in H_∞ Filter based SLAM.

Before presenting the main results, we analyze the feasibility condition for H_∞ Filter-SLAM.

Theorem 1. Consider (1)-(4) and (6). The filter solution is exist if it agree with the feasibility condition of $\gamma^2 > R$ and $\gamma^{-2} > P_0$ such that the solution are satisfying $P_0^{-1} > H^T R^{-1} H - \gamma^{-2} I$ and $1 > H^T H > \gamma^{-2} - P_0$.

Proof. As $P_0 > 0$, and $\gamma > 0$, we deduce that from (14), $P_0^{-1} > H_A^T R_A^{-1} H_A - \gamma^{-2} I$. To reveal this criteria clearly, consider 1-D SLAM(a robot with a single coordinate system observing landmarks. Eventually, we have $P_0^{-1} > H_A^T R_A^{-1} H_A - \gamma^{-2}$. By using this equation, by H.Ahmad et.al (2010) it is easy to verify that if $\gamma^2 > R$, then for 1-D SLAM the following must be satisfied.

$$\begin{aligned} H_A^T R_A^{-1} H_A &> \gamma^{-2} - P_0 \\ H_A^T H_A &> (\gamma^{-2} - P_0) R \end{aligned}$$

and $H_A^T H_A < 1$. Furthermore, by the above expression, it is easy to understand that in order to achieve above result, $\gamma^{-2} > P_0$ must be guaranteed in each observation.

Notice that unlike P.Bolzern et.al (1999), additional conditions are required. Now we are ready to examine the sufficient condition for convergence for each cases.

3.1 Case 1: $P_{0v} << P_{0m}$

The preceding section explains that FIM is used to interpret H_∞ Filter behavior in each update with comparison to the EKF. Hence, (14) is utilize to reveal its significance in pertaining the influence of design parameters to H_∞ Filter.

If $P_{0v} << P_{0m}$, then we assume that $P_{0m}^{-1} \to 0$. Thereby,

$$\Omega = \begin{bmatrix} P_{0v}^{-1} + H_A^T R_A^{-1} H - \gamma^{-2} I_n & -H^T R_A^{-1} A \\ -A^T R_A^{-1} H_A & A^T R_A^{-1} A - \gamma^{-2} I_n \end{bmatrix} \tag{16}$$

Notice that, the diagonal elements are essential for designer to obtain some sufficient conditions in H_∞ Filter. Moreover, bear in mind that Ω must always preserves at least a Positive semidefinite matrix(PsD) in each observation. These two facts are vital to secure a reliable estimation in H_∞ Filter.

Proposition 1. Given $\gamma > 0$. For a case of a robot that has more confidence about its initial state than the landmarks state, γ is affected by the initial state covariance, process and measurement noises and its selection must satisfying below properties.

$$\gamma > \sqrt{\frac{1}{P_{0\theta}^{-1} + R_A^{-1}}} \tag{17}$$

$$\gamma > \sqrt{\frac{R_A(dx^2+dy^2)^2}{(P_{0x}+Q_x)^{-1} R_A (dx^2+dy^2)^2 + dx^4 + dx^2 dy^2 + dy^2}} \tag{18}$$

$$\gamma > \sqrt{\frac{R_A(dx^2+dy^2)^2}{(P_{0y}+Q_y)^{-1} R_A (dx^2+dy^2)^2 + dy^4 + dy^2 dx^2 + dx^2}} \tag{19}$$

where Q_x, Q_y are the associated process noises for each robot x, y coordinates.

Proof. First the diagonal elements are observes. This lead us to analyze two elements of $P_{0v}^{-1} + H_A^T R_A^{-1} H - \gamma^{-2} I_n$ and $A^T R_A^{-1} A - \gamma^{-2} I_n$. The former element can substantially explains the later element. This is shown by the following calculations.

$$\begin{aligned} & P_{0v}^{-1} + H_A^T R_A^{-1} H - \gamma^{-2} I_n \\ &= \begin{bmatrix} P_{0\theta}^{-1} & 0 \\ 0 & P_{0xy}^{-1} \end{bmatrix} + \begin{bmatrix} -e^T \\ -A^T \end{bmatrix}^T R_A^{-1} [-e \quad A] - \gamma^{-2} I_n \\ &= \begin{bmatrix} P_{0\theta}^{-1} + e^T R_A^{-1} e - \gamma^{-2} & -e^T R_A^{-1} A \\ -A^T R_A^{-1} e & P_{0xy}^{-1} + A^T R_A^{-1} A - \gamma^{-2} I_n \end{bmatrix} \end{aligned} \tag{20}$$

where $P_{0\theta}$ and P_{0xy} are the initial robot state covariance about its angle and x, y position. Let P_{0x}, P_{0y} defines each x, y robot initial state covariances. Note that both diagonal elements must preserves PsD in each observation. Robot heading is the primary factor in SLAMS.Huang et.al (2007). Hence, it is analyze differently with other terms. As each diagonal matrix elements must at least a PsD, then for the robot heading angle covariance $P_{0\theta}$, require the following to be satisfied.

$$\begin{aligned} P_{0\theta}^{-1} + R_A^{-1} - \gamma^{-2} &> 0 \\ \gamma^2 &> \frac{1}{P_{0\theta}^{-1} + R_A^{-1}} \end{aligned} \tag{21}$$

The second diagonal element is check.

$$\begin{aligned} A^T R_A^{-1} A &= \begin{bmatrix} \frac{dx}{r} & -\frac{dy}{r^2} \\ \frac{dy}{r} & -\frac{dx}{r^2} \end{bmatrix} R_A^{-1} \begin{bmatrix} \frac{dx}{r} & \frac{dy}{r} \\ -\frac{dy}{r^2} & -\frac{dx}{r^2} \end{bmatrix} \\ &= \begin{bmatrix} (\frac{dx^2}{r^2} + \frac{dy^2}{r^4}) R_A^{-1} & (\frac{dxdy}{r^2} + \frac{dydx}{r^4}) R_A^{-1} \\ (\frac{dydx}{r^2} + \frac{dxdy}{r^4}) R_A^{-1} & (\frac{dy^2}{r^2} + \frac{dx^2}{r^4}) R_A^{-1} \end{bmatrix} \end{aligned}$$

As we consider a case of robot which has some degree of confidence about its initial location, we substitute above equation into the second diagonal term of (20). Subsequently, this approach lead us to the following expression.

$$P_{0x}^{-1} + \left[\frac{dx^2}{r^2} + \frac{dy^2}{r^4}\right] R_A^{-1} - \gamma^{-2} \geq 0 \tag{22}$$

$$P_{0y}^{-1} + \left[\frac{dy^2}{r^2} + \frac{dx^2}{r^4}\right] R_A^{-1} - \gamma^{-2} \geq 0 \tag{23}$$

Then we propose that γ must satisfy the following two conditions for estimation.

$$\gamma^2 > \frac{R_A(dx^2+dy^2)^2}{P_{0x}^{-1}R_A(dx^2+dy^2)^2 + dx^4 + dx^2dy^2 + dy^2} \tag{24}$$

$$\gamma^2 > \frac{R_A(dx^2+dy^2)^2}{P_{0y}^{-1}R_A(dx^2+dy^2)^2 + dy^4 + dy^2dx^2 + dx^2} \tag{25}$$

These results significantly describes that it is difficult to obtain an appropriate γ due to nonlinearities of robot movement and noises. Nevertheless, as each prediction also embraces the process noise, we concur that bigger process noise required bigger γ.

Next we examine the PsD characteristics in each FIM update. By this view, we find that it gives us a proper selection of γ.

Theorem 2. Given $\gamma > 0$ and *Theorem 1* is satisfied. If a the robot initial state covariance is very small than the initial landmarks state, then γ is choose to satisfy the following equations.

(1) $\gamma > \sqrt{R}$
(2) $\gamma > \sqrt{P_{0v}}$

If else, the updated state error covariance exhibit *F.E.T.*

Proof. From the properties of PsD, the determinant of the matrix must be nonnegative. This fact is use to obtain some criteria for γ selection. The determinant of (14) describes that

$$(P_{0v}^{-1} + H_A^T R_A^{-1} H_A - \gamma^{-2} I)(A^T R_A^{-1} A - \gamma^{-2} I) - H_A^T R_A^{-1} A^T R_A^{-1} H > 0 \tag{26}$$

The above nonlinear equation is difficult to explain the effect of γ in each update. We propose the analysis in linear 1-D SLAM to visualize γ influences. In 1-D SLAM, the determinant eventually become as stated below.

$$\begin{aligned}(P_{0v}^{-1} + R_A^{-1} - \gamma^{-2})(R_A^{-1} - \gamma^{-2}) - R_A^{-2} > 0 \\ P_{0v}^{-1}R_A^{-1} - \gamma^{-2}P_{0v}^{-1} - 2\gamma^{-2}R_A^{-1} + \gamma^{-4} > 0 \\ \gamma^{-4} - (2P_{0v}^{-1} + R_A^{-1})\gamma^{-2} + P_{0v}^{-1}R^{-1} > 0\end{aligned} \tag{27}$$

where $H_A = [-1 \quad 1]$ and $A = 1$. Furthermore, it is easily understood that as $P_{0v}, R_A > 0$, then the following are achieved.

$$\begin{aligned}&\gamma^{-4} - (2P_{0v}^{-1} + R_A^{-1})\gamma^{-2} + P_{0v}^{-1}R^{-1} \\ &\quad < \gamma^{-4} - (P_{0v}^{-1} + R_A^{-1})\gamma^{-2} + P_{0v}^{-1}R^{-1} \\ &\quad = (\gamma^{-2} - P_{0v}^{-1})(\gamma^{-2} - R_A^{-1}) > 0\end{aligned} \tag{28}$$

We now understand that there exist two different cases with two respective conditions.

(1) $\gamma > \sqrt{R}$ and $\gamma > \sqrt{P_{0v}}$
(2) $\gamma < \sqrt{R}$ and $\gamma < \sqrt{P_{0v}}$

However, condition (2) is unlikely to happen. This is due to by analyzing (14), this condition can yield a negative definite matrix. Therefore, condition (1) is apparently the solution for this case. More over, from above we explicitly identified the relationship between γ, initial state covariance and measurement noise. To add more, the process noise is also influencing γ selection as it is included in each state covariance prediction step.

It is known that according to the literatures, the robot angle act as an important factor to be considered in SLAM problemS.Huang et.al (2007). As been proposed by *Theorem 1* and *Proposition 1* in this paper, the designer must ensure $P_0 > R^{-1}$ and (21) are fulfilled. γ is selected such that by incrementally increasing its value corresponding to the value given by *Proposition 1* and *Theorem 1* to obtain the best solution. Now, we proceed to examine the next case for SLAM.

3.2 Case 2: $P_{0v} = P_{0m}$

This condition is the appropriate situation for an actual SLAM problem. It is obvious that, if a robot is arbitrarily put in an unknown environment, then it does not have information about its initial location even though is being equipped with high accuracy sensors. Such a situation presumes an uniform distribution for both robot and landmarks belief. We now proposed the following theorem to analyze the estimation behavior for H_∞ Filter based SLAM.

Theorem 3. Given $\gamma > 0$ and *Theorem 1* is satisfied. There is a γ that gives a best solution to SLAM which satisfy the following if and only if both robot and landmarks initial state covariance are very big such that robot does not have any prior information about its initial position.

$$\gamma > \sqrt{\frac{R}{1-\sqrt{R}}} \tag{29}$$

Proof. Consider that both robot and landmarks initial state covariances are too big. By referring to the previous case, the determinant of the updated state covariance for a robot observing landmarks at point A yield below equation.

$$(P_{0v}^{-1} + H_A^T R_A^{-1} H_A - \gamma^{-2} I)(P_{0m}^{-1} A^T R_A^{-1} A - \gamma^{-2} I) - H_A^T R_A^{-1} A A^T R_A^{-1} H_A > 0 \tag{30}$$

To simplify this analysis, we consider again 1-D SLAM problem. For convenience, let $P_{0v} = P_{0m}$. By this assumption, (29) drive us to the following equation.

$$(P_0^{-1} + R_A^{-1} - \gamma^{-2})^2 - R_A^{-2} > 0 \tag{31}$$

After calculations and some arrangements, we obtain that

$$\gamma^{-4} - 2\gamma^{-2}(P_0^{-1} + R_A^{-1}) + P_0^{-2} + 2P_0^{-1}R_A^{-1} > 0 \tag{32}$$

Consider about above equation and the fact that $P_0 >> 0 (P_0^{-1} \to 0)$. Hence, by through factorization, we arrived in γ that yield

$$\gamma > \sqrt{\frac{R}{1-\sqrt{R}}} \tag{33}$$

Remark that, process noise still slightly effect the estimation if it is too big. If such conditions occurred, then γ must be tune carefully to achieve a desired outcomes. Refer back to the filter algorithm, H_∞ Filter estimation should be same to EKF if γ is set to be very big. Related to this fact, we propose a condition of

γ where H_∞ Filter has better performance than EKF. If $F.E.T$ is observable, then γ must be increase to obtain good result. This is the common step in H_∞ Filtering which finally approximating the same estimation behavior to EKF. The next section identify and evaluate clearly about our result.

4. SIMULATION RESULTS

The results obtained above are examined through a simulation. We consider a small environment which have the parameter describes in Table 1. We also assume that the robot exteroceptive sensors can observes its surrounding and the process noise are small. The robot is assigned to move in some direction while doing observations. Landmarks are also assume to be point landmarks and are stationary. We compare the estimation results between H_∞ Filter and EKF in SLAM regarding map construction, state error covariance update and RMSE for each cases that have been analyzed in the preceding section. Note that the process noise are kept consistently very small for both cases.

Table 1. Simulation Parameters

Sampling Time, T	$0.1[s]$
Process noise,Q	1×10^{-7}
Observation noise, $R_{\theta_i}, R_{distance_i}$	$R_{\theta_i}=0.002, R_{distance_i}=0.002$
Robot Initial Covariance *Pvv*	1×10^{-2}
Landmarks Initial Covariance *Pmm*	1×10^{4}

Figs.3-5 illustrates the simulation results for case 1 whenever the mobile robot has confidence about its initial position in comparison to the landmarks $P_{0v} << P_{0m}$. Based on these figures, it is observable that the estimation of H_∞ Filter outperform EKF. The H_∞ Filter convergence is also smaller than EKF. The robot path estimation has sufficiently provides a meaningful results where H_∞ Filter is better than EKF. Evaluation about the RMSE for the robot position also guaranteed that H_∞ Filter has smaller error than EKF. These result are perceived if and only if the condition of $\gamma > \sqrt{R}$ as proposed in our theoretical analysis in the previous section.

On the other hand, Figs. 6-8 shows the results of case 2 of initial covariances of 1×10^4 for both robot and landmarks states. We consistently have the same performance as described by Fig.3-5. Robot can estimate its current path and location with some level of certainty. The uncertainties of estimation proved that H_∞ Filter still surpassing EKF performance. The RMSE evaluation about the robot path also contributes the same characteristics which also agree that H_∞ Filter can provide a better solution in SLAM problem if and only if $\gamma > \sqrt{R}$ is satisfied in each observations.

However, if the condition of $\gamma > \sqrt{R}$ is not fulfilled, then the estimation become erroneous as explained in Fig.9. The position of landmarks and robot are diversely located in the environment. EKF performs better in this situation.

Even though it seems that we must ensure $\gamma > \sqrt{R}$, remark that initial state covariance and process noise have the possibilities to influence the estimation. Bigger initial state covariance and process noise contributes to bigger selection of γ especially for case A. Nevertheless, we can conclude that all of the results

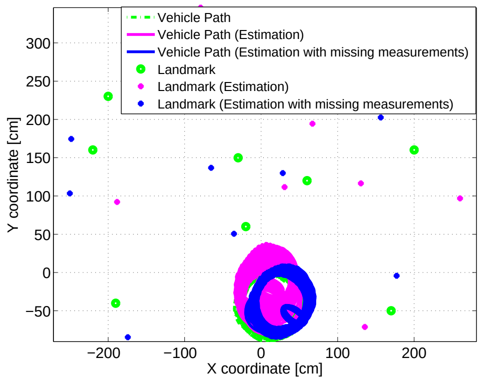

Fig. 3. Case 1: Robot localization and map building performance between H_∞ Filter-EKF

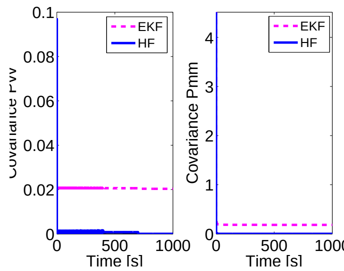

Fig. 4. Case 1: Updated state error covariance between H_∞ Filter-EKF

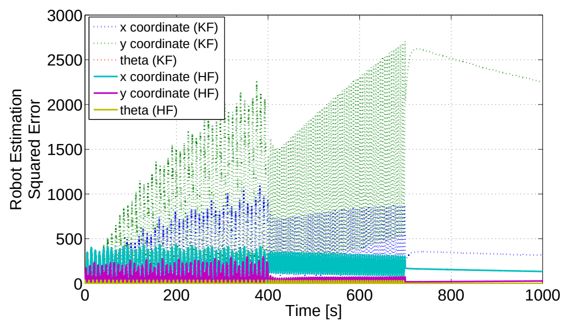

Fig. 5. Case 1: Robot estimation error performance between H_∞ Filter-EKF

sufficiently supports our analysis. Observing case B where the robot angle show that it require γ to at least satisfy $\gamma > \sqrt{R}$. It is known that the robot angle play an important role for estimation purposes and the difference between both filter performance is observable via Fig.5.

Even more, H_∞ Filter has guaranteed that the its estimation surpassed EKF even when in a gaussian noise environment with an appropriate selection of γ and parameters design. Besides, our results supports H.Ahmad et.al (2009) analysis as the state error covariance update converge almost to zero in the estimation. To add more, we also found that the EKF estimation becomes more inconsistent as the initial state covariance become bigger. Even if in this condition, HF still preserves better estimation. To conclude, H_∞ Filter-SLAM is one of the competitive solution for SLAM especially for bigger initial state covariance and non-gaussian noise environment.

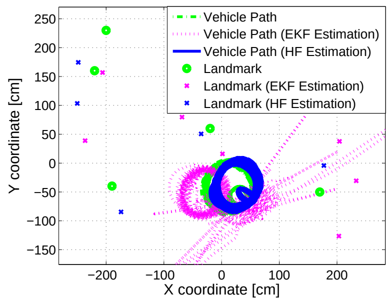

Fig. 6. Case 2: SLAM performance between H_∞ Filter-EKF

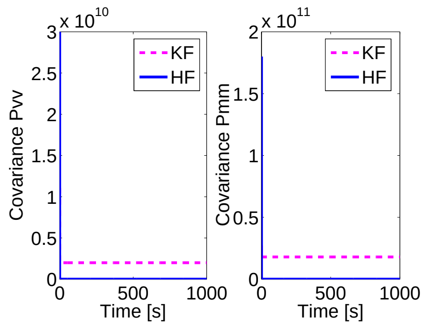

Fig. 7. Case 2: Updated state error covariance between H_∞ Filter-EKF

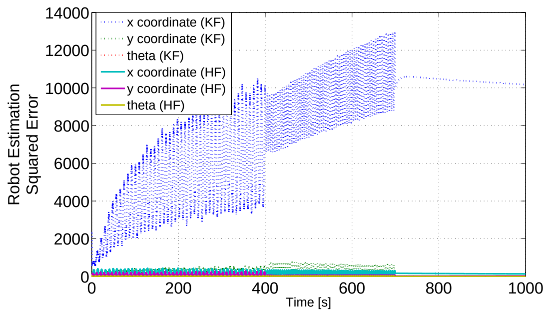

Fig. 8. Case 2: Robot estimation error performance between H_∞ Filter-EKF

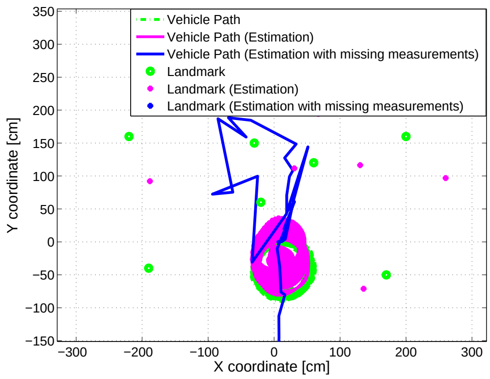

Fig. 9. H_∞ Filter divergence about estimation in comparison to EKF approach

5. CONCLUSION

We have already shown by theoretical analysis and experimental evaluations that H_∞ Filter is one of the candidates for SLAM especially for an environment with unknown noise characteristics. Given by two cases, we suggest that in general, the measurement noise must be less than γ^2 for a system which posses smaller process noise. Further attention is required if both initial state covariance and process noise are big which consequently demand bigger γ selection for the whole system to operate efficiently. However, to sufficiently achieve an expected performance in H_∞ Filter, designer must ensure that they satisfies the above given conditions in their system design regarding the conditions of initial state covariance, process and measurement noises distributions.

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