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Entropic cosmology is a recently proposed cosmological model that aims to explain the accelerated expansion of the Universe. In this study, we examine a nonadiabatic expansion of the universe in entropic cosmology, from a thermodynamics viewpoint. We derive the continuity (conservation) equation from the first law of thermodynamics, assuming the nonadiabatic expansion caused by the entropy and temperature on the horizon of the universe. Using the continuity equation, we reformulate entropic cosmology. The simple model proposed here agrees well with supernova data which take into account the Planck 2013 results.

KEYWORDS: cosmology, entropy, nonadiabatic process, thermodynamics

1. Introduction

To explain the accelerated expansion of the Universe, we usually assume an additional energy component called 'dark energy'. For example, famous ΛCDM models assume cold dark matter (CDM) and a cosmological constant Λ relating to dark energy. Alternatively, Easson *et al.* [1] have recently proposed that an entropic-force term should be added to the Friedmann–Lemaître acceleration equation, without introducing new fields [2]. In the entropic-force scenario, called 'entropic cosmology,' the additional entropic-force term is derived from the usually neglected surface terms on the horizon of the universe in the gravitational action, assuming that the horizon has an entropy and a temperature [1]. However, in entropic cosmology, the entropy can increase during the evolution of the universe as if it were a non-adiabatic-like (hereafter nonadiabatic) process [3]. Accordingly, we examine a nonadiabatic expansion of the universe in entropic cosmology, from a thermodynamics viewpoint. In this study, we derive the continuity equation from the first law of thermodynamics, taking into account the nonadiabatic process caused by the entropy and temperature on the horizon. Using the obtained continuity equation, we reformulate the modified Friedmann and acceleration equations, and propose a simple model. In the present paper, typical results studied in Ref. [3] are reconsidered and discussed using the recent Planck 2013 data [4].

2. Modified Friedmann and acceleration equations for entropic cosmology

In this section, we give a brief review of entropic cosmology [1–3]. We consider a homogeneous, isotropic, and spatially flat universe, and examine the scale factor $a(t)$ at time t in the Friedmann–Lemaître–Robertson–Walker metric. For entropic cosmology, Koivisto *et al.* [2] have summarized the modified Friedmann and acceleration equations given by

$$\left(\frac{\dot{a}(t)}{a(t)}\right)^2 = H(t)^2 = \frac{8\pi G}{3}\rho(t) + \alpha_1 H(t)^2 + \alpha_2 \dot{H}(t), \tag{1}$$

$$\frac{\ddot{a}(t)}{a(t)} = \dot{H}(t) + H(t)^2 = -\frac{4\pi G}{3}(1+3w)\rho(t) + \beta_1 H(t)^2 + \beta_2 \dot{H}(t), \tag{2}$$

where the Hubble parameter $H(t)$ is defined by $H(t) \equiv (da/dt)/a(t) = \dot{a}(t)/a(t)$. Note that we neglect high-order terms for quantum corrections relating to the inflation of the early universe [3]. G and $\rho(t)$ are the gravitational constant and the mass density of cosmological fluids, respectively. w represents the equation of state parameter for a generic component of matter, which is given as $w = p(t)/(\rho(t)c^2)$, where c and $p(t)$ are the speed of light and the pressure of cosmological fluids, respectively. For the matter- and radiation-dominated universes, w is 0 and 1/3, respectively. The four coefficients α_1, α_2, β_1, and β_2 are dimensionless constants [2]. The H^2 and $\dot{H}$ terms with the dimensionless constants correspond to the additional driving terms, in which we assume an entropy and a temperature on the horizon of the universe due to the information holographically stored there [1]. Coupling [$(1+3w)\times$ Eq. (1)] with [$2\times$ Eq. (2)] and rearranging, we obtain

$$\dot{H} = \frac{dH}{dt} = -C_1 H^2 \quad \text{where} \quad C_1 = \frac{3(1+w) - \alpha_1(1+3w) - 2\beta_1}{2 - \alpha_2(1+3w) - 2\beta_2}. \tag{3}$$

As examined in Ref. [3], we can solve Eq. (3), assuming a single-fluid-dominated universe. For example, the luminosity distance d_L is obtained as

$$\left(\frac{H_0}{c}\right) d_L = \begin{cases} \frac{1+z}{C_1-1}\left[1-(1+z)^{-C_1+1}\right] & (C_1 \neq 1), \\ (1+z)\ln(1+z) & (C_1 = 1), \end{cases} \tag{4}$$

where z is the redshift defined by $z \equiv (a_0/a) - 1$. H_0 and a_0 are the present values of the Hubble parameter and the scale factor, respectively. The results for $C_1 = 2$, 1.5, 1, and 0 are consistent with those for the radiation-, matter-, empty, and Λ-dominated universes, respectively [3].

3. Reformulation of entropic cosmology

In this section, we derive the continuity equation from the first law of thermodynamics, assuming a nonadiabatic expansion of the universe [3]. Using the obtained continuity equation, we reformulate entropic cosmology.

The first law of thermodynamics in an expanding (or contracting) universe [3] is written as

$$dQ = dE + pdV = \left[\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right)\right] c^2 \left(\frac{4\pi}{3} r_s(t)^3\right) dt, \tag{5}$$

where dQ is the heat flow across a region, and dE and dV are changes in the internal energy E and volume V of the region, respectively. $r_s(t)$ is a proper radius. If we assume adiabatic (and isentropic) processes, then dQ is 0, that is, $dQ = TdS = 0$, where S and T represent the entropy and temperature, respectively. However, in this paper, we assume a nonadiabatic process given by $dQ = TdS \neq 0$. To calculate TdS, we employ the Hubble radius as the preferred screen [1] for entropic cosmology. Accordingly, the proper radius r_s included in Eq. (5) is replaced by the Hubble radius r_H. The Hubble radius is given as

$$r_H = \frac{c}{H} \quad \text{and therefore} \quad \dot{r}_H = -\frac{H\dot{H}}{c^2} r_H^3. \tag{6}$$

For entropic cosmology, we assume that the Hubble horizon has an associated entropy S and an approximate temperature T [1–3]. The entropy and temperature are written as

$$S = \frac{k_B c^3}{\hbar G}\frac{A_H}{4} \quad \text{and} \quad T = \frac{\hbar H}{2\pi k_B} \times \gamma = \frac{\hbar}{2\pi k_B}\frac{c}{r_H}\gamma, \tag{7}$$

where k_B, $\hbar$, and A_H are the Boltzmann constant, the reduced Planck constant, and the surface area of a sphere with the Hubble radius r_H, respectively [1]. The reduced Planck constant is defined by $\hbar \equiv h/(2\pi)$, where h is the Planck constant. It should be noted that γ is a non-negative free parameter and is of the order of $O(1)$; typically $\gamma \sim \frac{3}{2\pi}$ or $\frac{1}{2}$ [3].

Using Eq. (7) and $dA_H/dt = d(4\pi r_H^2)/dt = 8\pi r_H \dot{r}_H$, we calculate TdS as

$$TdS = T \times \frac{k_B c^3}{4\hbar G}\frac{dA_H}{dt}dt = \frac{\hbar}{2\pi k_B}\frac{c}{r_H}\gamma \times \frac{k_B c^3}{4\hbar G}(8\pi r_H \dot{r}_H)dt = \gamma \frac{c^4}{G}\dot{r}_H dt. \tag{8}$$

Replacing r_s by r_H, substituting Eqs. (5), (6), and (8) into $dQ = dE + pdV = TdS$, and rearranging, we obtain the modified continuity equation as

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) = -\gamma\left(\frac{3}{4\pi G}H\dot{H}\right). \tag{9}$$

The non-zero right-hand side of Eq. (9) is related to a nonadiabatic expansion of the universe. Using an effective description similar to bulk viscous cosmology [5], the non-zero term can be cancelled in appearance, i.e., $\dot{\rho} + 3(\dot{a}/a)(\rho + p'/c^2) = 0$, where p' is an effective pressure [3]. However, in this approach, it is necessary to examine an effective description for entropic cosmology; this will be discussed in our next paper [6]. Of course, if H is constant, Eq. (9) is the continuity equation for an adiabatic (isentropic) process. Note that parameters for the entropy, e.g., Tsallis' entropic parameter [7], may be required for calculating TdS.

As shown in Eqs. (1) and (2), entropic force terms include four dimensionless constants α_1, α_2, β_1, and β_2. We determine most of the dimensionless constants using two continuity equations [3]. The first continuity equation is Eq. (9), while the second continuity equation can be derived from the modified Friedmann and acceleration equations. As examined in Ref [3], from Eqs. (1) and (2), the second continuity equation is expressed as

$$\dot{\rho} + 3\frac{\dot{a}}{a}(1+w)\rho = \frac{3}{4\pi G}\left[(-\alpha_1 - \alpha_2 + \beta_2)H\dot{H} + (-\alpha_1 + \beta_1)H^3 - \frac{\alpha_2}{2}\ddot{H}\right]. \tag{10}$$

We can expect that the two modified continuity equations, Eqs. (9) and (10), are consistent with each other. Therefore, when $\ddot{H}$, H^3, and $H\dot{H}$ are not 0, the constants reduce to $\alpha_2 = 0$, $\beta_1 = \alpha_1$, and $\beta_2 = \alpha_1 - \gamma$, respectively. Moreover, we assume $\alpha_1 = \gamma$ for the simple model [3]. Consequently, the simple model is summarized as

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho + \alpha_1 H^2 = \frac{8\pi G}{3}\rho + \gamma H^2, \tag{11}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(1+3w)\rho + \alpha_1 H^2 + (\alpha_1 - \gamma)\dot{H} = -\frac{4\pi G}{3}(1+3w)\rho + \gamma H^2. \tag{12}$$

Note that γ should be determined from a different viewpoint, as discussed later.

4. Comparison with both ΛCDM models and supernova data points

In this section, by considering the luminosity distance d_L, we compare the simple model with both ΛCDM models and supernova data points. To this end, we consider the matter-dominated universe given by $w = 0$. Moreover, γ is set to be $3/(2\pi)$. Therefore, C_1 is $\frac{3}{2}(1 - \frac{3}{2\pi}) = 0.7838\cdots$. The coefficient $3/(2\pi)$ was deduced from the surface term order [1] without using a fitting method. (Alternatively, C_1 can be determined through fitting with a fine-tuned standard ΛCDM model or supernova data points. For example, C_1 is approximately

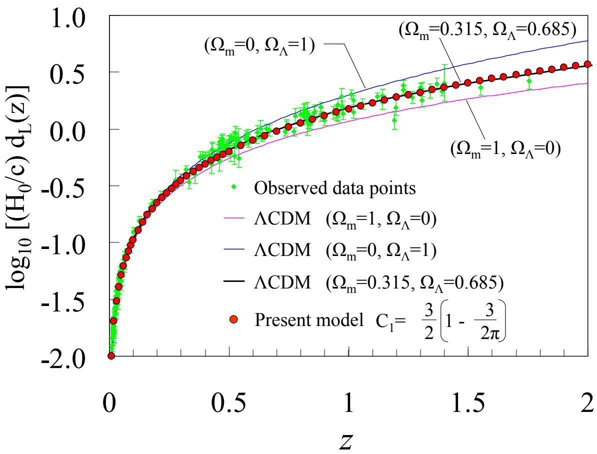

Fig. 1. (Color online). Dependence of luminosity distance d_L on redshift z. Supernova data points are taken from Ref. [8], where H_0 is set to be 67.3 km/s/Mpc based on the Planck 2013 results [4].

calculated as $C_1 \approx 0.78$ using a fitting method [6].) For the ΛCDM models, we assume a spatially flat universe, neglecting the density parameter for the radiation [3]. The universe in which $(\Omega_m, \Omega_\Lambda) = (0.315, 0.685)$ is a fine-tuned standard ΛCDM model, which takes into account the recent Planck 2013 best fit values [4]. Ω_m and Ω_Λ represent the density parameters for matter and Λ, respectively. As shown in Fig. 1, the simple model agrees well with supernova data points and the fine-tuned standard ΛCDM model. Hence, it has been shown that the simple model can describe the present accelerating universe successfully without adding the cosmological constant or dark energy.

5. Conclusions

From a thermodynamics viewpoint, we have examined the nonadiabatic expansion of the universe in entropic cosmology, and reconsidered typical results from Ref. [3]. We have derived the modified continuity equation from the first law of thermodynamics. Using the obtained continuity equation, we have reformulated the modified Friedmann and acceleration equations. By using the luminosity distance as a means of comparison, it is clearly shown that the simple model agrees well with both the fine-tuned standard ΛCDM model and the supernova data, which take into account the Planck 2013 results.

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