Dynamic Control of Multifingered Hands for Pivoting Operation

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Dynamic Control of Multifingered Hands for Pivoting Operation

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Abstract

In this paper, we propose a dynamic control method of multifingered hands for pivoting an object in contact with the environment. This pivoting operation is often observed when a human moves a large or heavy object such as furniture on the floor. Different from the conventional manipulation of the object by only fingers, the characteristics of the pivoting operation is that we can use the reaction force from the environment. By using this reaction force, we can expect the magnitude of the forces applied to the object by the fingers is smaller than the conventional manipulation of the object by only fingers. In this paper, taking this characteristics of the reaction force into consideration, we propose a dynamic control method for pivoting. To verify our approach, simulation results are also presented.

1. Introduction

Many researchers have studied the manipulation of an object which contacts with an environment such as a table. While the environment constrains the motion of the object, we can use the reaction force from the environment in the manipulation of the object. Then, we can expect that the needed number of the fingers to manipulate the object and the magnitudes of contact forces applied by the fingers are smaller than the conventional manipulation of the object by only fingers such as Li et al.[1], Cole et al.[2], and Yokokohji et al.[3]. In this paper, the goal is to establish a control method for forming force closure[4] grasp and manipulating the object in contact with the environment by small contact forces even when we cannot form force closure grasp by only fingers. Especially, we are interested in pivoting operation. Pivoting operation is a rotation of the object around an axis through a contact point between the object and the environment. This manipulation is often observed when a human moves a large or heavy object such as furniture. Aiyama et al[5] have proposed the quasi-static motion planning for manipulating a rectangular object, but we cannot apply the planning to a general object. On the other hand, some manipulations similar to the pivoting operation have been studied. Han et al.[7] proposed a control method for the quasi-static manipulation of the spherical object on a plane by one finger which has 6 D.O.F. and a flat fingertip. Trinkle et al.[8] quasi-statically analyzed the manipulation of lifting up the object on the support. In another paper[9], Trinkle et al. proposed a quasi-static manipulation planning for enveloping the slippery work piece on the palm and manipulating it. But the dynamic control method for these manipulations has been studied only by a few researchers (for example, Yoshikawa et al.[6], but, the control method is proposed under the assumption that we can form force closure grasp by only fingers). In this paper, we propose a dynamic control method of multifingered hands for pivoting an object in contact with the environment.

This paper is organized as follows. In section 2, we describe the target system of this paper for pivoting operation, and in section 3, we develop a dynamic control method for pivoting operation. In section 4, we consider the derivation of desirable magnitude of internal forces by which the magnitude of the contact forces of the fingers can become as smallas possible. To verify our approach, we show simulation results in section 5.

2. Target System

The target system of this paper is shown in **Fig.1**. This system consists of two fingers and a rigid object. each finger has three joints and a spherical soft fingertip, the object contacts the environment at one vertex, and each contact surface is smooth. The given task is to pivot the object in contact with the environment at the vertex by the two fingers along with a desired trajectory. We make the following assumption for the analysis of the soft fingertips; (i) the contact area is so small that the contact between the soft fingertip and the object is regarded as a point contact, (ii) the tangential frictional force and frictional twist/spin moment around the contact normal are independent and they are approximated by the Coulomb model, and (iii) the energy dissipation due to the fingertip deformation is negligible.

3. Control Scheme

In this section, we propose the control scheme based on the formulations by Li et al.[1], Cole et al.[2], Yokokohji et al.[3], and Yoshikawa et al.[6]. Note that we permit a rotational slip at the contact point between the fingers and the object.

3.1. Kinematics Constraints

In this subsection, we formulate the kinematic constraints between the fingers and the object and between the environment and the object. As shown in **Fig.**2, Σ_R, Σ_B, $\Sigma_{Fi}(i=1,2)$, and $\Sigma_{C_i}(i=0,1,2)$ denote a reference coordinate frame, an object coordinate frame placed at the gravity center, a fingertip coordinate frame placed at the center of the spherical fingertip of the finger i, and a frame fixed on the object surface at the contact point C_i, respectively. Let the position of the origin and the orientation of Σ_a with respect to Σ_c (Σ_R) be given by

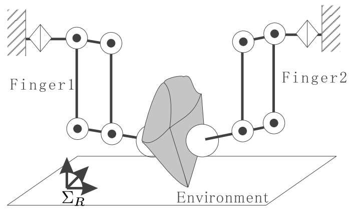


Figure 1: Target System

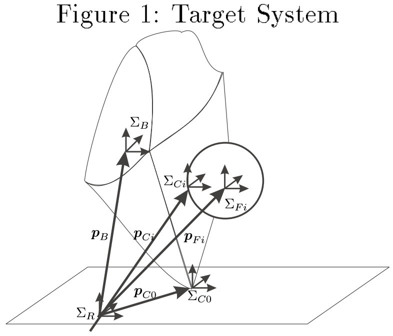


Figure 2: Objet, fingers, and Environment

${}^c\boldsymbol{p}_a(\boldsymbol{p}_a)$ and ${}^c\boldsymbol{R}_a$ $(\boldsymbol{R}_a)$, respectively. Let a vector directing from the origin of Σ_a to the origin of Σ_b with respect to Σ_c (Σ_R) be given by ${}^c\boldsymbol{p}_{ab}$ $(\boldsymbol{p}_{ab})$.

First, the constraint at the fixed contact point between the environment and the object is given by

$$\boldsymbol{H}_{C0}\boldsymbol{D}_{B0}\begin{pmatrix}\dot{\boldsymbol{p}}_B\\ \boldsymbol{\omega}_B\end{pmatrix}=\boldsymbol{H}_{C0}\begin{pmatrix}\dot{\boldsymbol{p}}_{C0}\\ \boldsymbol{\omega}_{C0}\end{pmatrix}=\dot{\boldsymbol{p}}_{C0}=\boldsymbol{0} \tag{1}$$

where

$$\boldsymbol{D}_{Bi} = \begin{pmatrix}\boldsymbol{I}_3 & -[(\boldsymbol{R}_B{}^B\boldsymbol{p}_{BCi})\times]\\ \boldsymbol{O}_3 & \boldsymbol{I}_3\end{pmatrix} \tag{2}$$

$$\boldsymbol{H}_{C0} = [\boldsymbol{I}_3 \quad \boldsymbol{O}_3] \tag{3}$$

Here, $\boldsymbol{\omega}_B$ and $\boldsymbol{\omega}_{Ci}$ denote an angular velocities of the object(Σ_B) and Σ_{Ci}, $[\bullet\times]$ denotes a skew symmetric equivalent to the cross product operation, and $\boldsymbol{I}_i$ and $\boldsymbol{O}_i$ denote the i-order identity and null matrices, respectively. Here, instead of the angular velocity of the object $\boldsymbol{\omega}_B$, we use roll, pitch and yaw angles for expressing the object orientation. The relation between $\boldsymbol{\omega}_B(=\boldsymbol{\omega}_{C0})$ and the velocity of roll, pitch and yaw angles, $\dot{\boldsymbol{\phi}}_B(=\dot{\boldsymbol{\phi}}_{C0})$, is given by $\boldsymbol{\omega}_B=\boldsymbol{T}_r\dot{\boldsymbol{\phi}}_B$ where $\boldsymbol{T}_r(\boldsymbol{\phi}_B)$ denotes the matrix for the transformation. Then, from (1), we get

$$\boldsymbol{D}_{B0}\begin{pmatrix}\dot{\boldsymbol{p}}_B\\ \boldsymbol{\omega}_B\end{pmatrix}=\begin{pmatrix}\dot{\boldsymbol{p}}_{C0}\\ \boldsymbol{\omega}_{C0}\end{pmatrix}=\boldsymbol{T}\begin{pmatrix}\boldsymbol{0}\\ \dot{\boldsymbol{\phi}}_{C0}\end{pmatrix} \tag{4}$$

where $\boldsymbol{T}=diag[\boldsymbol{I}_3\ \boldsymbol{T}_r(\boldsymbol{\phi}_{C0})]$ ("diag" means a block diagonal matrix).

Second, since the contact between the finger and the object is a soft-finger type contact, the constraint at the contact point between the finger $i(i=1,2)$ and the object is given by

$$\begin{aligned}\boldsymbol{H}_{Ci}\boldsymbol{D}_{Bi}\begin{pmatrix}\dot{\boldsymbol{p}}_B\\ \boldsymbol{\omega}_B\end{pmatrix}&=\boldsymbol{H}_{Ci}\begin{pmatrix}\boldsymbol{v}_{Ci}\\ \boldsymbol{\omega}_{Ci}\end{pmatrix}\\ &=\boldsymbol{H}_{Ci}\boldsymbol{D}_{Fi}\begin{pmatrix}\dot{\boldsymbol{p}}_{Fi}\\ \boldsymbol{\omega}_{Fi}\end{pmatrix}\end{aligned} \tag{5}$$

where

$$\boldsymbol{D}_{Fi} = \begin{pmatrix}\boldsymbol{I}_3 & -[(\boldsymbol{R}_{Fi}{}^{Fi}\boldsymbol{p}_{FiCi})\times]\\ \boldsymbol{O}_3 & \boldsymbol{I}_3\end{pmatrix} \tag{6}$$

$$\boldsymbol{H}_{Ci} = \begin{cases}\begin{pmatrix}\boldsymbol{I}_3 & \boldsymbol{O}_3\\ \boldsymbol{0} & \boldsymbol{n}_{Ci}^T\end{pmatrix} & \text{(not spinning)}\\ [\boldsymbol{I}_3 \quad \boldsymbol{O}_3] & \text{(spinning)}\end{cases} \tag{7}$$

Here, $\boldsymbol{v}_{Ci}$ denotes a vector of the contact point velocity, where the components of the contact point movement due to the rolling are excluded, and $\boldsymbol{\omega}_{Fi}$ denotes an angular velocity of Σ_{Fi}.

Finally, the relation between the fingertip velocity of finger i and the joint velocities of finger i is given by

$$\begin{pmatrix}\dot{\boldsymbol{p}}_{Fi}\\ \boldsymbol{\omega}_{Fi}\end{pmatrix}=\boldsymbol{J}_{Fi}\dot{\boldsymbol{q}}_i \tag{8}$$

where $\boldsymbol{J}_{Fi}$ denotes the Jacobian matrix of finger i and $\boldsymbol{q}_i$ denotes the joint vector of finger i.

From (4) (5) (8), we get

$$\begin{aligned}\boldsymbol{H}_{Ci}\boldsymbol{D}_{Bi}\boldsymbol{D}_{B0}^{-1}\boldsymbol{T}\begin{pmatrix}\boldsymbol{0}\\ \dot{\boldsymbol{\phi}}_{C0}\end{pmatrix}&=\boldsymbol{H}_{Ci}\begin{pmatrix}\boldsymbol{v}_{Ci}\\ \boldsymbol{\omega}_{Ci}\end{pmatrix}\\ &=\boldsymbol{H}_{Ci}\boldsymbol{D}_{Fi}\boldsymbol{J}_{Fi}\dot{\boldsymbol{q}}_i=\boldsymbol{J}_{CFMi}\dot{\boldsymbol{q}}_i\end{aligned} \tag{9}$$

3.2. Dynamics of Fingers and Object

In this subsection, we formulate the dynamics of the fingers and the object. Let $\boldsymbol{t}_B$ be the resultant force and moment applied to the object by the fingers at the origin of the frame Σ_B, and $\boldsymbol{f}_f$ be the force applied to the environment by the object at the contact point (note that $-\boldsymbol{f}_f$ expresses the reaction force). Then, from (1), the resultant force applied by both the fingers and the environment is given by $\boldsymbol{t}_B-\boldsymbol{D}_{B0}^T\boldsymbol{H}_{C0}^T\boldsymbol{f}_f$. So, the dynamics of the object is given by

$$\boldsymbol{M}_B\begin{pmatrix}\ddot{\boldsymbol{p}}_B\\ \dot{\boldsymbol{\omega}}_B\end{pmatrix}+\boldsymbol{h}_B=\boldsymbol{t}_B-\boldsymbol{D}_{B0}^T\boldsymbol{H}_{C0}^T\boldsymbol{f}_f \tag{10}$$

where $\boldsymbol{M}_B$ denotes an inertia tensor of the object and $\boldsymbol{h}_B$ denotes centrifugal, Coriolis, and gravitational forces.

Next, the dynamics of the fingers is given by

$$\boldsymbol{M}_F(\boldsymbol{q})\ddot{\boldsymbol{q}}+\boldsymbol{h}_F(\boldsymbol{q},\dot{\boldsymbol{q}})=\boldsymbol{\tau}-\boldsymbol{J}^T\boldsymbol{f}_S \tag{11}$$

where $\boldsymbol{\tau}=col\ [\boldsymbol{\tau}_1\ \boldsymbol{\tau}_2]\ \in R^6$ ("col" means a column vector or matrix formed by the following vector or matrix) ($\boldsymbol{\tau}_i\in R^3$ denotes a joint torque of finger i), $\boldsymbol{f}_S=col\ [\boldsymbol{f}_{S1}\ \boldsymbol{f}_{S2}]$ ($\boldsymbol{f}_{Si}$ denotes the force and the moment applied to the object by finger i at the contact point C_i expressed by a vector representing the magnitudes of the components in the directions expressed by a column vector of $\boldsymbol{H}_{Ci}^T$ in (7)), $\boldsymbol{M}_F=diag\ [\boldsymbol{M}_{F1}\ \boldsymbol{M}_{F2}]$($\boldsymbol{M}_{Fi}$ denotes an inertia matrix of finger i), $\boldsymbol{h}_F=diag\ [\boldsymbol{h}_{F1}\ \boldsymbol{h}_{F2}]$($\boldsymbol{h}_{Fi}$ denotes centrifugal, Coriolis, gravitational forces, and so on), and $\boldsymbol{J}=\boldsymbol{J}_{CFM}+\boldsymbol{J}_{sm}$ where $\boldsymbol{J}_{CFM}=diag\ [\boldsymbol{J}_{CFM1}\ \boldsymbol{J}_{CFM2}]$(refer to (9)) and

$\boldsymbol{J}_{sm} = diag\ [\boldsymbol{J}_{sm1}\ \boldsymbol{J}_{sm2}]$. Here, $\boldsymbol{J}_{smi}$ is represented by

$$\boldsymbol{J}_{smi}^T = \boldsymbol{J}_{Fi}^T \begin{pmatrix} \boldsymbol{O}_3 & \boldsymbol{O}_3 \\ \boldsymbol{N}_i & \boldsymbol{O}_3 \end{pmatrix} \boldsymbol{H}_{Ci}^T \tag{12}$$

$$\boldsymbol{N}_i = \alpha_i \mu_i \boldsymbol{n}_{Ci} \boldsymbol{n}_{Ci}^T \tag{13}$$

where $\boldsymbol{n}_{Ci}$ denotes the contact normal, $\alpha_i (= -1, 0, or 1)$ shows the direction of the frictional twist/spin moment at the contact point C_i, and μ_i is determined by the same law proposed by Yokokohji et al.[3] and denotes the dynamic rotational frictional coefficient of finger i if a rotational slip occurs at C_i. Note that $\boldsymbol{J}_{smi}^T \boldsymbol{f}_{Si}$ expresses a joint torque equivalent to the dynamic frictional twist/spin moment, $[\boldsymbol{O}_3\ \boldsymbol{N}_i]\boldsymbol{H}_{Ci}^T \boldsymbol{f}_{Si}$.

3.3. Contact Forces and Moment

In this subsection, we formulate the force and the moment applied to the object by the fingers at the contact points, $\boldsymbol{f}_S$, and the reaction force, $-\boldsymbol{f}_f$.

First, we formulate $\boldsymbol{f}_S$. If a rotational slip doesn't occur at C_i between finger i and the object, the contact force and moment $\boldsymbol{f}_{Si}$ has 4 D.O.F. On the other hand, finger i has 3 D.O.F. Then, we express $\boldsymbol{f}_S$ as follows

$$\boldsymbol{f}_S = \boldsymbol{J}\boldsymbol{J}^+ \boldsymbol{f}_S + (\boldsymbol{I} - \boldsymbol{J}\boldsymbol{J}^+)\boldsymbol{f}_S \triangleq \boldsymbol{J}\boldsymbol{f}_C + \tilde{\boldsymbol{J}}\tilde{\boldsymbol{f}}_C \tag{14}$$

where $\boldsymbol{f}_C \in \boldsymbol{R}^6$, $\tilde{\boldsymbol{f}}_C \in \boldsymbol{R}^2$, $\tilde{\boldsymbol{J}}$ is an arbitrary matrix which satisfies $\boldsymbol{J}^T\tilde{\boldsymbol{J}} = \boldsymbol{0}$, and $\boldsymbol{J}^+$ denotes the pseudo-inverse matrix of $\boldsymbol{J}$. Note that the former term in (14) denotes the controllable contact force and moment by the joint torques, and the latter term in (14) denotes the uncontrollable contact force and moment. Then, from (5) (14) and the principle of virtual work, the relation between $\boldsymbol{t}_B$ and $\boldsymbol{f}_S$ is given by

$$\boldsymbol{t}_B = \bar{\boldsymbol{A}}(\boldsymbol{J}\boldsymbol{f}_C + \tilde{\boldsymbol{J}}\tilde{\boldsymbol{f}}_C) \triangleq \boldsymbol{A}\boldsymbol{f}_C + \widetilde{\boldsymbol{A}}\tilde{\boldsymbol{f}}_C \tag{15}$$

$$\bar{\boldsymbol{A}} \triangleq (\boldsymbol{D}_B^T + \begin{pmatrix} \boldsymbol{O}_3 & \boldsymbol{O}_3 & \boldsymbol{O}_3 & \boldsymbol{O}_3 \\ \boldsymbol{N}_1 & \boldsymbol{O}_3 & \boldsymbol{N}_2 & \boldsymbol{O}_3 \end{pmatrix})\boldsymbol{H}_C^T \tag{16}$$

where $\boldsymbol{D}_B$= col $[\boldsymbol{D}_{B1}\ \boldsymbol{D}_{B2}]$, $\boldsymbol{H}_C$= diag $[\boldsymbol{H}_{C1}\ \boldsymbol{H}_{C2}]$. Note that the second term in (16) relates the dynamic rotational friction.

From (15), we get

$$\boldsymbol{f}_C = \boldsymbol{A}^+(\boldsymbol{t}_B - \widetilde{\boldsymbol{A}}\tilde{\boldsymbol{f}}_C) + \boldsymbol{\Phi}\tilde{\boldsymbol{f}} \tag{17}$$

where $\boldsymbol{\Phi}$ is an arbitrary matrix which satisfies $\boldsymbol{A}\boldsymbol{\Phi} = \boldsymbol{0}$, $\tilde{\boldsymbol{f}}$ express the magnitude of the component in the direction expressed by a column vector of $\boldsymbol{\Phi}$, and $\boldsymbol{A}^+$ denotes the pseudo-inverse matrix of $\boldsymbol{A}$.

Next, we formulate $\boldsymbol{f}_f$. Now we hope that the magnitude of the contact force and moment applied to the object by the fingers can be as small as possible. But, we can control only the former term in (14). Then, we consider the following problem.

$$\min \quad \boldsymbol{f}_C^T \boldsymbol{J}^T \boldsymbol{J} \boldsymbol{f}_C \tag{18}$$

$$\text{subj. to.} \quad \boldsymbol{G}\begin{pmatrix} \boldsymbol{f}_C \\ \boldsymbol{f}_f \end{pmatrix} = \hat{\boldsymbol{t}}_B$$

where

$$\boldsymbol{G} \triangleq (\boldsymbol{A} \quad -\boldsymbol{D}_{B0}^T \boldsymbol{H}_{C0}^T)$$

$$\hat{\boldsymbol{t}}_B \triangleq \boldsymbol{h}_B + \boldsymbol{M}_B\{\boldsymbol{D}_{B0}^{-1}\boldsymbol{T}\begin{pmatrix} \boldsymbol{0} \\ \ddot{\phi}_{C0} \end{pmatrix} (\dot{\boldsymbol{D}}_{B0}^{-1}\boldsymbol{T} + \boldsymbol{D}_{B0}^{-1}\dot{\boldsymbol{T}})\begin{pmatrix} \boldsymbol{0} \\ \dot{\phi}_{C0} \end{pmatrix}\}$$

Note that the constraint in the problem is in order to complement the dynamics of the object by $\boldsymbol{J}\boldsymbol{f}_C$ and $\boldsymbol{f}_f$. By using (17), if we solve the problem with respect to $\boldsymbol{f}_f$, $\tilde{\boldsymbol{f}}$, and $\tilde{\boldsymbol{f}}_C$, we can get $\tilde{\boldsymbol{f}}_C = \boldsymbol{0}$, $\tilde{\boldsymbol{f}} = \boldsymbol{0}$, and the following equation.

$$\begin{pmatrix} \boldsymbol{f}_C \\ \boldsymbol{f}_f \end{pmatrix} = \tilde{\boldsymbol{G}}\hat{\boldsymbol{t}}_B \tag{19}$$

where

$$\tilde{\boldsymbol{G}} = \begin{pmatrix} \boldsymbol{A}^+(\boldsymbol{I}_6 - \boldsymbol{D}_{B0}^T\boldsymbol{H}_{C0}^T\boldsymbol{B}_1) \\ -\boldsymbol{B}_1 \end{pmatrix}$$

$$\boldsymbol{B}_1 = \boldsymbol{B}_3\boldsymbol{B}_4^+(\boldsymbol{I}_6 - \boldsymbol{D}_{B0}^T\boldsymbol{H}_{C0}^T\boldsymbol{B}_3\boldsymbol{B}_2^T\boldsymbol{J}\boldsymbol{A}^+) + \boldsymbol{B}_3(\boldsymbol{A}^+)^T\boldsymbol{J}^T\boldsymbol{J}\boldsymbol{A}^+$$

$$\boldsymbol{B}_2 = \boldsymbol{J}\boldsymbol{A}^+\boldsymbol{D}_{B0}^T\boldsymbol{H}_{C0}^T$$

$$\boldsymbol{B}_3 = (\boldsymbol{B}_2^T\boldsymbol{B}_2)^{-1}\boldsymbol{H}_{C0}\boldsymbol{D}_{B0}$$

$$\boldsymbol{B}_4 = (\boldsymbol{I} - \boldsymbol{A}\boldsymbol{A}^+)\boldsymbol{D}_{B0}^T\boldsymbol{H}_{C0}^T\boldsymbol{B}_3(\boldsymbol{I} - \boldsymbol{A}\boldsymbol{A}^+)^T$$

3.4. Internal Force

In this subsection, we consider an internal force for forming force closure grasp. In this paper, we consider not only an internal force between the fingers but also an internal force between the finger and the environment. Then, if let $[(\boldsymbol{J}\boldsymbol{f}_C^I)^T \quad (\boldsymbol{f}_f^I)^T]^T$ be the internal force, the internal force satisfy the following equation.

$$\boldsymbol{G}\begin{pmatrix} \boldsymbol{f}_C^I \\ \boldsymbol{f}_f^I \end{pmatrix} = \boldsymbol{0} \tag{20}$$

Now, we set the component of the internal force $[(\boldsymbol{f}_C^I)^T \quad (\boldsymbol{f}_f^I)^T]^T$ as follows.

$$\begin{pmatrix} \boldsymbol{f}_C^I \\ \boldsymbol{f}_f^I \end{pmatrix} = diag(\boldsymbol{J}_{t1}^{-1}\ \boldsymbol{J}_{t2}^{-1}\ \boldsymbol{I}_3)\tilde{\boldsymbol{\Lambda}}\boldsymbol{k} = \boldsymbol{\Lambda}\boldsymbol{k} \tag{21}$$

$$\tilde{\boldsymbol{\Lambda}} = \begin{pmatrix} \boldsymbol{e}_{12} + \zeta_{11}\boldsymbol{e}_n & \boldsymbol{e}_{10} + \zeta_{12}\boldsymbol{e}_n & \zeta_{13}\boldsymbol{e}_n \\ \boldsymbol{e}_{21} + \zeta_{21}\boldsymbol{e}_n & \zeta_{22}\boldsymbol{e}_n & \boldsymbol{e}_{20} + \zeta_{23}\boldsymbol{e}_n \\ \Sigma_{i=1}^2 \zeta_{i1}\boldsymbol{e}_n & \boldsymbol{e}_{10} + \Sigma_{i=1}^2 \zeta_{i2}\boldsymbol{e}_n & \boldsymbol{e}_{20} + \Sigma_{i=1}^2 \zeta_{i3}\boldsymbol{e}_n \end{pmatrix}$$

where $\boldsymbol{e}_{ij}$ denote an unit vector which directs from the contact point C_i to the contact point C_j, $\boldsymbol{k}$=$[k_1\ k_2\ k_3]^T$ denote the magnitude of the components of the internal force in the directions expressed by a column vector of $\boldsymbol{\Lambda}$, $\boldsymbol{e}_n$ denotes an arbitrary vector orthogonal to both $\boldsymbol{n}_{C1}$ and $\boldsymbol{n}_{C2}$, and $\boldsymbol{J}_{ti} (i = 1, 2)$ is a component of $\boldsymbol{J}$ expressing the following equation.

$$\boldsymbol{J}^T = \begin{pmatrix} \boldsymbol{J}_{t1}^T & \boldsymbol{J}_{r1}^T & \boldsymbol{O}_3 & \boldsymbol{O}_3 \\ \boldsymbol{O}_3 & \boldsymbol{O}_3 & \boldsymbol{J}_{t2}^T & \boldsymbol{J}_{r2}^T \end{pmatrix} \boldsymbol{H}_C^T$$

We determine ζ_{ij} $(i = 1, 2\ j = 1, 2, 3)$ in (21) by substituting (21) to (20) . Then, $[(\boldsymbol{J}\boldsymbol{f}_C^I)^T \ (\boldsymbol{f}_f^I)^T]^T$ given by

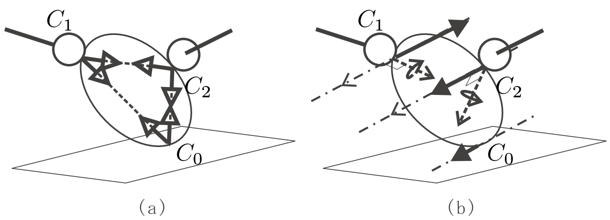


Figure 3: Internal Forces

(21) represent the internal force. This internal forces includes both the term(white arrows in **Fig.3**(a)) given by a conventional method to determine the internal forces in the manipulation of an object by three fingers and the term (black arrows in **Fig.3**) which include the static/dynamic frictional twist/spin moment at the contact point $C_i (i = 1, 2)$ and the force whose direction is $\boldsymbol{e}_n$ and which cancels the moment.

3.5. Controller for Pivoting

In this subsection, we derive a controller for pivoting operation.

From (1) (9) (10) (11) (15), the dynamics of the system for pivoting is given by

$$\begin{aligned}
\boldsymbol{W}\boldsymbol{x} &= \boldsymbol{b} = col[\boldsymbol{b}_1\ \boldsymbol{b}_2\ \boldsymbol{b}_3\ \boldsymbol{b}_4] \qquad (22)\\
\boldsymbol{W} &= \begin{pmatrix} \boldsymbol{M}_f & \boldsymbol{O}_6 & \boldsymbol{J}^T\boldsymbol{J} & \boldsymbol{O} & \boldsymbol{O} \\ \boldsymbol{O}_6 & \boldsymbol{M}_B & -\boldsymbol{A} & \tilde{\boldsymbol{A}} & \boldsymbol{D}_{B0}^T\boldsymbol{H}_{C0}^T \\ \boldsymbol{J}_{CFM} & -\boldsymbol{H}_C\boldsymbol{D}_B & \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{H}_{C0}\boldsymbol{D}_{B0} & \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O}_3 \end{pmatrix}\\
\boldsymbol{b}_1 &= \boldsymbol{\tau} - \boldsymbol{h}_f,\ \boldsymbol{b}_2 = -\boldsymbol{h}_B,\ \boldsymbol{b}_4 = -\boldsymbol{H}_{C0}\dot{\boldsymbol{D}}_{B0}\begin{pmatrix} \dot{\boldsymbol{p}}_B \\ \boldsymbol{\omega} \end{pmatrix}\\
\boldsymbol{b}_3 &= (\dot{\boldsymbol{H}}_C\boldsymbol{D}_B + \boldsymbol{H}_C\dot{\boldsymbol{D}}_B)\begin{pmatrix} \dot{\boldsymbol{p}}_B \\ \boldsymbol{\omega} \end{pmatrix} - \dot{\boldsymbol{J}}_{CFM}\dot{\boldsymbol{q}}
\end{aligned}$$

where $\boldsymbol{x} = [\ddot{\boldsymbol{q}}^T\ \ddot{\boldsymbol{p}}_B^T\ \dot{\boldsymbol{\omega}}_B^T\ \boldsymbol{f}_C^T\ \tilde{\boldsymbol{f}}_C^T\ \boldsymbol{f}_f^T]^T$.

By supposing the desired value of $\boldsymbol{x}$ is $\boldsymbol{x}_d$=$[\ddot{\boldsymbol{q}}_d\quad \ddot{\boldsymbol{p}}_{Bd}\ \dot{\boldsymbol{\omega}}_{Bd}\ \boldsymbol{f}_{Cd}\ \tilde{\boldsymbol{f}}_C^*\ \boldsymbol{f}_f^*]^T$ (note that $\tilde{\boldsymbol{f}}_C^*$ cannot be controlled and that $\boldsymbol{f}_f^*$ include both the desired value related with $\boldsymbol{f}_{Cd}$ and uncontrollable value related with $\tilde{\boldsymbol{f}}_C^*$), we can get the following controller.

$$\begin{aligned}
\boldsymbol{\tau} &= \boldsymbol{M}_f\ddot{\boldsymbol{q}}_d + \boldsymbol{h}_f + \boldsymbol{J}^T\boldsymbol{J}\boldsymbol{f}_{Cd} \qquad (23)\\
\ddot{\boldsymbol{q}}_d &= \boldsymbol{J}_{CFM}^+\{\boldsymbol{H}_C\boldsymbol{D}_B\begin{pmatrix} \ddot{\boldsymbol{p}}_{Bd} \\ \dot{\boldsymbol{\omega}}_{Bd} \end{pmatrix} - \dot{\boldsymbol{J}}_{CFM}\dot{\boldsymbol{q}}\\
&\quad + (\dot{\boldsymbol{H}}_C\boldsymbol{D}_B + \boldsymbol{H}_C\dot{\boldsymbol{D}}_B)\begin{pmatrix} \dot{\boldsymbol{p}}_B \\ \boldsymbol{\omega} \end{pmatrix}\} \qquad (24)\\
\boldsymbol{f}_{Cd} &= [\boldsymbol{I}_6\quad \boldsymbol{O}_3]\{\boldsymbol{\Lambda}\boldsymbol{u}_I\\
&\quad + \tilde{\boldsymbol{G}}(\boldsymbol{M}_B\begin{pmatrix} \ddot{\boldsymbol{p}}_{Bd} \\ \dot{\boldsymbol{\omega}}_{Bd} \end{pmatrix} + \boldsymbol{h}_B)\} \qquad (25)\\
\begin{pmatrix} \ddot{\boldsymbol{p}}_{Bd} \\ \dot{\boldsymbol{\omega}}_{Bd} \end{pmatrix} &= \boldsymbol{D}_{B0}^{-1}\boldsymbol{T}\begin{pmatrix} \boldsymbol{0} \\ \boldsymbol{u}_B \end{pmatrix}\\
&\quad + (\dot{\boldsymbol{D}}_{B0}^{-1}\boldsymbol{T} + \boldsymbol{D}_{B0}^{-1}\dot{\boldsymbol{T}})\begin{pmatrix} \boldsymbol{0} \\ \dot{\phi}_{C0} \end{pmatrix} \qquad (26)
\end{aligned}$$

where $\boldsymbol{u}_B$ and $\boldsymbol{u}_I$ are new inputs.

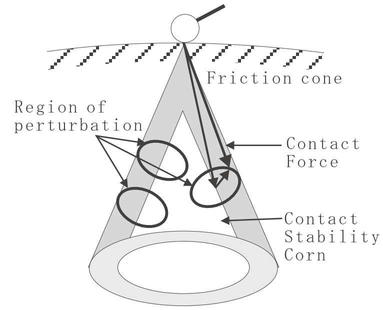


Figure 4: Contact Stability Corn

By applying the controller to the system (22), we get $\boldsymbol{W}(\boldsymbol{x}_d - \boldsymbol{x}) = \boldsymbol{0}$. If $\boldsymbol{W}$ is nonsingular, we can get $\boldsymbol{x}_d = \boldsymbol{x}$. Then, we get $\ddot{\boldsymbol{\phi}}_{C0} = \boldsymbol{u}_B$ from (26) and $\boldsymbol{k} = \boldsymbol{u}_I$ from (25).

Letting $\boldsymbol{\phi}_{C0d}$ and $\boldsymbol{k}_d$ be the desired trajectory of $\boldsymbol{\phi}_{C0}$ and $\boldsymbol{k}$ respectively , $\boldsymbol{u}_B$ and $\boldsymbol{u}_I$ are given by the following servo controller

$$\begin{aligned}
\boldsymbol{u}_B &= \ddot{\boldsymbol{\phi}}_{C0d} + \boldsymbol{K}_V(\dot{\boldsymbol{\phi}}_{C0d} - \dot{\boldsymbol{\phi}}_{C0})\\
&\quad + \boldsymbol{K}_P(\boldsymbol{\phi}_{C0d} - \boldsymbol{\phi}_{C0}) \qquad (27)\\
\boldsymbol{u}_I &= \boldsymbol{k}_d + \boldsymbol{K}_I\int_0^t(\boldsymbol{k}_d - \boldsymbol{k})dt' \qquad (28)
\end{aligned}$$

where $\boldsymbol{K}_P$, $\boldsymbol{K}_V$ and $\boldsymbol{K}_I$ are gain matrices. Then, if $\boldsymbol{W}$ is nonsingular, the object orientation error $\boldsymbol{e}_P = \boldsymbol{\phi}_{C0d} - \boldsymbol{\phi}_{C0}$ and the force error $\boldsymbol{e}_f = \boldsymbol{k}_d - \boldsymbol{k}$ satisfy the following equations

$$\begin{aligned}
\ddot{\boldsymbol{e}}_P + \boldsymbol{K}_V\dot{\boldsymbol{e}}_P + \boldsymbol{K}_P\boldsymbol{e}_P &= \boldsymbol{0} \qquad (29)\\
\dot{\boldsymbol{e}}_f + \boldsymbol{K}_I\boldsymbol{e}_f &= \boldsymbol{0} \qquad (30)
\end{aligned}$$

Thus, with appropriate $\boldsymbol{K}_P$, $\boldsymbol{K}_V$ and $\boldsymbol{K}_I$, the actual object orientation $\ddot{\boldsymbol{\phi}}_{C0}$ and the component of the internal forces $\boldsymbol{k}$ will converge to the desired trajectories asymptotically.

4. Nonlinear Programming

Introducing of internal force, the contact force and moment applied by fingers formulated in section 3 isn't as small as possible. Then, in this section, we derive a magnitude of the component of the internal force by which the contact force and moment applied by fingers can become as small as possible.

We consider the following nonlinear programming.

$$\begin{aligned}
&\min && h(\boldsymbol{k}) = \boldsymbol{f}_C^T\boldsymbol{J}^T\boldsymbol{J}\boldsymbol{f}_C \qquad (31)\\
&subj.\ to. && \boldsymbol{l}(\boldsymbol{k}) = \boldsymbol{V}(\begin{pmatrix} \boldsymbol{f}_S \\ \boldsymbol{f}_f \end{pmatrix} + \boldsymbol{f}_{ext}) \leq \boldsymbol{0}
\end{aligned}$$

Here, $\boldsymbol{l} \leq \boldsymbol{0}$ express the translational frictional constraints which approximately represent the friction cone at the contact point C_i as the polyhedral convex cone with L_i faces [10], where $\boldsymbol{V}$=diag$[\boldsymbol{V}_1\ \boldsymbol{V}_2\ \boldsymbol{V}_0]^T$, $\boldsymbol{V}_i$=col$[(a_k^i)^T]$ $(k = 1, 2, \cdots, L_i)$, and a_k^i denotes an outward normal vector of the kth face of the polyhedral convex cone. Note that $\boldsymbol{f}_{ext}$ in the constraint in the programming denotes the contact force and moment related with an unexpected perturbation of input and the uncontrollable contact force and moment. we determine

$\boldsymbol{f}_{ext}$ by using the concept of Contact Stability Corn proposed by Nakamura et al.[12](**Fig.4**).

The contact force and moment related with an unexpected perturbation of input $\boldsymbol{f}_{SI}$ can be expressed by

$$\begin{aligned} \boldsymbol{f}_{SI} &= diag[\boldsymbol{J} \quad \boldsymbol{I}_3]\tilde{\boldsymbol{G}}\boldsymbol{M}_B\boldsymbol{D}_{B0}^{-1}\boldsymbol{T}\begin{pmatrix}\boldsymbol{O}_3\\ \boldsymbol{I}_3\end{pmatrix}\Delta\ddot{\boldsymbol{\phi}}_{C0} \\ &\triangleq [\hat{\boldsymbol{G}}_1^T \quad \hat{\boldsymbol{G}}_2^T \quad \hat{\boldsymbol{G}}_0^T]^T\Delta\ddot{\boldsymbol{\phi}}_{C0} \end{aligned} \quad (32)$$

where $\Delta\ddot{\boldsymbol{\phi}}_{C0}$ denotes the perturbation. On the other hand, the contact force and moment related with uncontrollable contact force and moment $\boldsymbol{f}_{UC}$ can be expressed by

$$\boldsymbol{f}_{UC} = \begin{pmatrix}\tilde{\boldsymbol{J}}\\ (\boldsymbol{D}_{B0}^T\boldsymbol{H}_{C0}^T)^{+}\tilde{\boldsymbol{A}}\end{pmatrix}\tilde{\boldsymbol{f}}_C \triangleq \begin{pmatrix}\hat{\boldsymbol{J}}_1\\ \hat{\boldsymbol{J}}_2\\ \hat{\boldsymbol{J}}_0\end{pmatrix}\tilde{\boldsymbol{f}}_C \quad (33)$$

Note that f_{UC0} in (33) expresses the contact force between the object and the environment occurred by uncontrollable contact force and moment applied by the fingers where $\boldsymbol{f}_{SIi}$ and $\boldsymbol{f}_{UCi}$ denotes the components of $\boldsymbol{f}_{SI}$ and $\boldsymbol{f}_{UC}$ at C_i respectively.

Now, the desirable internal force makes $\boldsymbol{f}_{Si}+\boldsymbol{f}_{SIi}+\boldsymbol{f}_{UCi}$ can be in the approximated friction corn as well as the contact force and moment applied by the fingers can be as small as possible. Then, we consider

By supposing a maximum magnitude of the perturbation $\Delta\ddot{\boldsymbol{\phi}}_{C0}$ is $\Delta\phi$ and a maximum magnitude of norm of uncontrollable contact force and moment $\tilde{\boldsymbol{f}}_C$ is f_{CM}, the sets of $\boldsymbol{f}_{SIi}$ and $\boldsymbol{f}_{UCi}$ are given by

$$\{\boldsymbol{f}_{SIi} | \boldsymbol{f}_{SIi}^T\boldsymbol{f}_{SIi} \leq \Delta\phi^2\hat{\boldsymbol{G}}_i^T\hat{\boldsymbol{G}}_i\} \quad (34)$$

$$\{\boldsymbol{f}_{UCi} | \boldsymbol{f}_{UCi}^T\boldsymbol{f}_{UCi} \leq f_{CM}^2\tilde{\boldsymbol{J}}_i^T\tilde{\boldsymbol{J}}_i\} \quad (35)$$

From this equation, we can determine the component of $\boldsymbol{f}_{ext}$, $\boldsymbol{f}_{exti}$ as follows.

$$\boldsymbol{f}_{exti} = \frac{\sqrt{\mu_{si}^2+1}}{\mu_{si}}(\Delta\phi\sigma_{Gimax} + f_{CM}\sigma_{Jimax})\boldsymbol{n}_{Ci}$$

where σ_{Gimax} and σ_{Jimax} denote the maximum singular value of $\hat{\boldsymbol{G}}_i$ and $\hat{\boldsymbol{J}}_i$ respectively.

This nonlinear programming (31) can be solved by using Kuhn-Tucker optimality conditions and linear complementarily problem [11].

The obtained $\boldsymbol{k}$ become a desirable component of the internal force.

5. Simulation

We show simulation results in this section to verify the validity of our approach. We did the following 3 kinds of simulations; 1) dynamic control with constant internal force 2) quasi-static control with constant internal force 3) dynamic control with obtained internal force by nonlinear programming. Quasi-static control is for the comparison and its scheme is given by neglecting the term related with both the acceleration and the velocity in the equation of motion of the fingers and by assuming the direction of the displacement of the objects coincides with the direction of the resultant force and moment applied to the origin of Σ_B in dynamic control scheme.

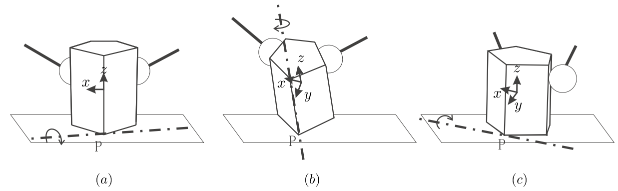


Figure 5: Transition of State

Suppose that the object is a regular prism whose bottom is a regular pentagon whose side is $0.05[m]$ length, whose height is $0.1[m]$ length, and whose weight is $0.5[kg]$. Each finger is a 3 D.O.F. parallel link robot (Fig.1) which is the same robot used in the experience in Yokokohji et al.[3], and each fingertip is a spherical soft fingertip with $0.018[m]$ radius. The initial configuration of the object with respect to the reference coordinate frame is $(0.[m]\ 0.[m]\ 0.05[m]\ 0.[rad]\ 0.[rad]\ 0.[rad])^T$ and the orientation of the object is expressed by roll, pitch, and yaw. The initial position of Σ_{F1} is $(-0.0162[m]\quad -0.0498[m]\quad 0.080[m])^T$ and the one of Σ_{F2} is $(\ -0.0162[m]\quad 0.0498[m]\quad 0.080[m]\ )^T$. We set each dynamic rotational frictional coefficient $0.002[m]$, each maximum static rotational frictional coefficient $0.0023[m]$, and each maximum static translational frictional coefficients 1.0. Servo gains are set to $\boldsymbol{K}_P = diag[\ 10000.\quad 10000.\quad 10000.](1/sec^2)$, $\boldsymbol{K}_V = diag[\ 200.\quad 200.\quad 200.](1/sec)$, and $\boldsymbol{K}_I = diag[\ 100.\quad 100.\quad 100.](1/sec)$ in dynamic control and $\boldsymbol{K}_P = diag[\ 3.425\quad 3.425\quad 3.425](Nm/rad)$, $\boldsymbol{K}_V = diag[\ 0.003425\quad 0.003425\quad 0.003425](Nmsec/rad)$, and $\boldsymbol{K}_I = diag[100.\quad 100.\quad 100.](1/s)$ in quasi-static control , and sampling time is set to $1.0[msec]$.

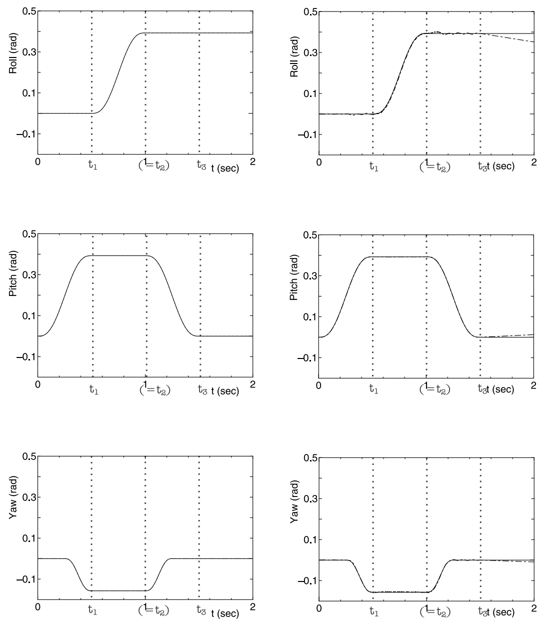
We consider the desired trajectory as follows; first, tilting (raising) the object around the vertex P(**Fig.5**(a)) until $(0\ \ \pi/8\ \ -\pi/20)^T[rad]$ for the interval from $t_0(=\ 0.0[sec])$ to $t_1(=\ 0.5[sec])$, then, rotating the object around the vertex P(**Fig.5**(b)) until $(\pi/8\ \ \pi/8\ \ -\pi/20)^T[rad]$ for the interval from $t_1(= 0.5[sec])$ to $t_2(= 1.[sec])$, finally, lowering the object around the vertex P(**Fig.5**(c)) until $(\pi/8\ 0\ 0)^T[rad]$ for the interval from $t_2(= 1.[sec])$ to $t_3(= 1.5[sec])$. And the fingers are removed form the object at $t_3(= 1.5[sec])$.

When we let the internal force constant, we set $\boldsymbol{k}_d$ =$[1.\quad 1.\quad 1.](N)$ in dynamic control and $\boldsymbol{k}_d$ =$[200.\ 200.\ 200.](N)$ in quasi-static control. On the other hand, when we use the nonlinear programming in section 4, we set the parameter as follows;

$$\Delta\phi = \begin{cases} =\|\ \ddot{\boldsymbol{\phi}}_{C0d}\ \| *0.1 & (\|\ \ddot{\boldsymbol{\phi}}_{C0d}\ \|> 10.) \\ = 1.[rad/sec^2] & (\|\ \ddot{\boldsymbol{\phi}}_{C0d}\ \|\leq 10.) \end{cases}$$

, $f_{UCM} = 0.1[N]$, and we approximate the friction cone as the polyhedral convex cone with 10 faces.

The result is shown in **Fig.6** and **Fig.7**. Fig.6(a) shows the actual object orientation (dot dash line) and the desired one (solid line) in dynamic control, and Fig.6(b) shows the ones in quasi-static control. From



(a)Results of Dynamic Control

(b)Results of Quasi-static Control

Figure 6: Simulation Result of Pivoting (orientation)

these figures, we can see that the actual object orientation converge to the desired orientation and that the deviation after t_3 in quasi-static control occurs because the velocity of the object doesn't converge to 0 at t_3. This result shows that dynamic control is more efficient than quasi-static control. Fig.7 shows the norm of contact force in case of dynamic control with constant internal force (solid line) and in case of dynamic control with the obtained internal force by the nonlinear programming (dot line). From these figures, we can see that the magnitude of the forces applied to the object by the fingers is smaller than by the environment, namely, we can manipulate the object by small contact force applied by the fingers by using the reaction force from the environment. we can also see that the contact force in case of using the obtained internal force by the nonlinear programming is smaller than in case of using constant internal force. So, the validity of the nonlinear programming can be shown.

6. Conclusions

In this paper, we have proposed a dynamic control method of soft-finger hands for pivoting an object in contact with the environment. The characteristics of the pivoting is that we can use a reaction force from the environment. By using this reaction force, we can expect the magnitude of the forces applied to the object by the fingers is smaller than the conventional manipulation of the object by only fingers. In this paper, taking this characteristics of the reaction force into consideration, we propose a dynamic control method for pivoting operation. To verify our approach, we have also presented the simulation results.

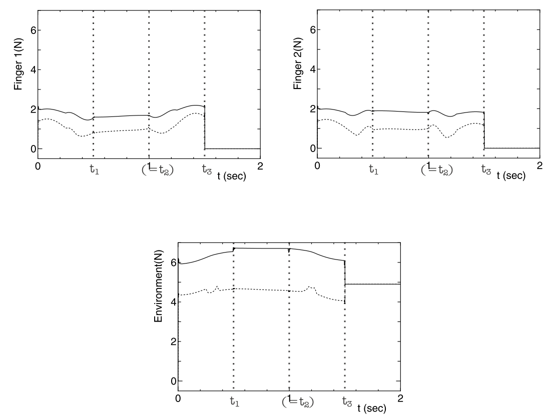


Figure 7: Simulation Result of Pivoting (norm of force)

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