Optimization of Grasping by Using a Required External Force Set

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Abstract

In this paper, we consider an optimization of grasping by using a required external force set. By using the set, we can not only deal with whatever a desired grasp is, such as force closure or equilibrium grasp, but also evaluate the magnitudes of the resistible external forces and moments. Then, we define an optimization problem from the viewpoint of decreasing the magnitudes of the contact forces required to resist the required external force, and show that we can solve the problem by using a branch-and-bound method. Lastly we present some numerical simulations to show the validity of our approach.

1 Introduction

Consider lifting up an object on a table by a robotic hand. If we don't grasp the object with appropriate contact positions between the object and the robotic hand, we may not be able to counteract the external force such as gravitational force and fail to lift up it. We must grasp an object with appropriate contact positions, in order to manipulate the object in a desired direction. Therefore, much attention has been attracted to search optimal grasp points on a grasped object [1]～[8].

In the above researches, there are 2 kinds of researches; 1) one is aimed at searching optimal grasp points or regions for counteracting gravitational force (namely, achieving equilibrium grasp) [1] [2], 2) the other is aimed at searching optimal grasp points or regions for achieving force closure [3]～[8]. Force closure indicates that if external force and moment in any arbitrary direction applied to a grasped object, we can counteract the external force and moment and restrain the motion of the object.

There exist an upper bound of the magnitude of the resistible external force and moment in force closure grasps. Then, the magnitude can be used as a criterion for the grasps. Li et al. [9] evaluated a volume of the largest task ellipsoid. It can be embedded in the set constructed by resultant forces and moments, which we can apply to a grasped object. But the computation of the volume is very complex. Then, it is hard to search optimal grasp points by using this criterion. On the other hand, researches about searching optimal grasp points, which minimize the magnitudes of the necessary contact forces to resist an external force and moment, has been done [4]～[7]. Also here, a problem arises: how to handle the difference between the unit of force and the one of moment. Then, some devices, to resolve the problem, have been done in these researches. Markenscoff et al. [4] considered the case where no external moment is applied to the object. Mirtich et al. [5] evaluated the external force and moment separately. Mangialardi et al. [6] considered the case where the external force or moment applied to the object individually. Wang et al. [7] regarded the magnitude of the resistible external force and moment as the magnitude of the contact force of the clamp.

In our previous paper, we optimized power grasp by using required external force set [10] [11]. This set consists of external forces and moments, which we must resist by contact forces applied by a robotic hand. The magnitude of the external force and moment contained in the set corresponds to the magnitude of the resistible external force and moment. When the origin is in the interior of the set, we can achieve force closure. When the dimension of the set is 1, we can achieve equilibrium grasp. Therefore we can deal with general desirable grasp including force closure and equilibrium grasp by using the set. Teichmann et al. [8] considered to search grasp points which minimize the number of the contact points needed to resist all external forces and moments contained in the set, not considering friction. But in their approach, we can deal with only force closure grasps.

In this paper, we search optimal grasp points which minimize the magnitudes of the contact forces required to resist all external forces and moments contained in the required external force set. We consider the case where an arbitrary shaped object is grasped by an arbitrary number of fingers with friction.

This paper is organized as follows. At first, we set the problem to solve. Then, we show an algorithm to solve the problem. Lastly, numerical examples are presented to show the effectiveness of our approach.

2 Problem Definition

In this section, we give a problem for optimal grasp. At first, we describe the target system, statics between a grasped object and a robotic hand, and the frictional constraints. Then, we define required external force set. Lastly, we give the problem for optimal grasp.

2.1 Target System

In this paper, we consider the case where an arbitrary shaped rigid object is grasped by m fingers of a robotic hand. We make the following assumptions: 1) Each finger makes a frictional point contact with the object at the fingertip, 2) The unique normal direction at each contact point can be obtained.

2.2 Constraints for Problem

At first, we describe the relation between the contact forces and the external force and moment applied to the object. We denote the set of candidate contact points by $\mathcal{C}$ and denote the combination of m selected contact points from $\mathcal{C}$ by

$$\mathcal{N}=\{\boldsymbol{p}_{C_{\mathcal{N}i}}\ (i=1,2,\cdots,m)|\boldsymbol{p}_{C_{\mathcal{N}i}}\in\mathcal{C}\} \tag{1}$$

where $\boldsymbol{p}_{C_{\mathcal{N}i}}\in\mathcal{R}^d(d=2/3$ in 2/3 dimensional space) represents the position of the ith contact point ($i=1,2,\cdots,m$). Let $\boldsymbol{f}_{\mathcal{N}i}\in\mathcal{R}^d$ be the contact force applied to the object at the ith contact point by the ith finger and let $\boldsymbol{w}\in\mathcal{R}^D(D=3/6$ in 2/3 dimensional space) be the external force and moment applied to the object(the origin of a frame fixed at the object). Then, the object can be in equilibrium if the following equation holds.

$$\Sigma_{i=1}^{m}\boldsymbol{G}_{\mathcal{N}i}\boldsymbol{f}_{\mathcal{N}i}=-\boldsymbol{w} \tag{2}$$

where $\boldsymbol{G}_{\mathcal{N}i}^T=(\boldsymbol{I}\ -[(\boldsymbol{p}_{C_{\mathcal{N}i}}-\boldsymbol{p}_o)\times]\ )\in\mathcal{R}^{d\times D}$ in 3 dimensional space. Here, $\boldsymbol{I}$ represents an identify matrix, $[\boldsymbol{a}\times]$ represents a skew symmetric matrix equivalent to the cross product operation($[\ \boldsymbol{a}\times]\boldsymbol{b}=\boldsymbol{a}\times\boldsymbol{b}$), $\boldsymbol{p}_o$ represents the origin of the frame fixed at the object.

Next, the frictional constraint at the ith($i=1,2,\cdots,m$) contact point is represented by

$$\mathcal{F}_{f\mathcal{N}i}=\{\boldsymbol{f}_{\mathcal{N}i}|\sqrt{t_{f_{\mathcal{N}i},1}^2+t_{f_{\mathcal{N}i},2}^2}\le\mu_{\mathcal{N}i}n_{f_{\mathcal{N}i}},n_{f_{\mathcal{N}i}}\ge0\} \tag{3}$$

in 3 dimensional space, where $n_{f_{\mathcal{N}i}}$ denotes the normal force component of $\boldsymbol{f}_{\mathcal{N}i}$, $t_{f_{\mathcal{N}i},1}$ and $t_{f_{\mathcal{N}i},2}$ denote the tangential force components of $\boldsymbol{f}_{\mathcal{N}i}$, and $\mu_{\mathcal{N}i}$ denotes the coefficient of maximum static friction at the ith contact point. In 2 dimensional space, we have only to set $t_{f_{\mathcal{N}i},2}=0$.

2.3 Required External Force Set

We define required external force set [10] [11] as follows.

Required External Force Set *We call a set, composed of all resultant forces and moments, which we can apply to the object, possible-generated force set $\mathcal{W}\subset\mathcal{R}^D$. Let $\mathcal{W}^-$ be $\{-\boldsymbol{w}|\boldsymbol{w}\in\mathcal{W}\}$. Required external force set $\mathcal{W}_R\subset\mathcal{R}^D$ is a set which $\mathcal{W}^-$ must contain.*

2.4 Problem for Optimal Grasp

Based on the above discussion, we define a problem to obtain an optimal grasp. In general, there exist an infinite number of combinations of contact points, whose $\mathcal{W}^-$ contains the given $\mathcal{W}_R$. We think it is suitable to minimize the magnitudes of the contact forces needed to resist the external force and moment. Now, let ϕ be the largest norm of contact force among all norms of all contact forces.

$$\phi=\max_i|\boldsymbol{f}_{\mathcal{N}i}|$$

We consider the following problem.

Problem for Optimal Grasp Find the combination of contact points $\mathcal{N}^*$ such that

$$\rho=\min_{\mathcal{N}\in\mathcal{S}}\ \max_{\boldsymbol{w}\in\mathcal{W}_R}\ \min_{\boldsymbol{f}_{\mathcal{N}i}\text{ satisfy (2),(3)}}\ \phi \tag{4}$$

where $\mathcal{S}$ denotes the set of the candidate $\mathcal{N}$. This problem is to search $\mathcal{N}$ which minimizes ϕ required to counteract $\boldsymbol{w}$ ($\in\mathcal{W}_R$).

3 Algorithm

In this section, we describe an algorithm to solve the problem (4). In order to solve the problem, we use a branch-and-bound method [12] [13] by representing the candidate contact points as discrete points. Note that we can deal with any object whose geometry is arbitrary complex, by using the discrete candidate contact points. The branch-and-bound method finds the optimal solution by enumerating the solutions of feasible subproblems into which we can partition the original problem. In the process, we cut the subproblems that we don't have to solve, by using a relaxed problem whose constraints relax the constraints of the subproblem. It makes the computational time reduced.

In order to use the branch-and-bound method, we make the 2 following assumptions: 1) the number of the candidate contact points on the object is finite n, 2) $\mathcal{W}_R$ can be expressed by a convex polyhedron composed of l vertexes.

From this assumption 1, the number of the candidate combinations of contact points becomes $C(n,m)$($C(n,m)$ means the number of the combinations where we select m from n). In this assumption

2, if $\mathcal{W}_R$ is not a convex polyhedron, we will define new $\mathcal{W}_R$ which is a convex polyhedral set which contains the original $\mathcal{W}_R$. Then, $\mathcal{W}_R$ can be represented by

$$\begin{aligned}\mathcal{W}_R \;\; = \;\; & \{\boldsymbol{w} = \Sigma_{j=1}^{l}\lambda_j \boldsymbol{w}_{vj},\ \Sigma_{j=1}^{l}\lambda_j = 1, \\ & \lambda_j \geq 0 (j = 1, 2, \cdots, l)\}\end{aligned} \tag{5}$$

where $\boldsymbol{w}_{vj}$ denotes the jth vertex of $\mathcal{W}_R$.

Now, we define some subproblems and a relaxed problem of the problem (4) at first, and then we describe the procedure of the algorithm.

3.1 Subproblems and Relaxed Problem

At first, we consider the case where we select some $\mathcal{N}_k$ from the $C(n, m)$ candidate combinations of contact points. Then, we obtain the following subproblem.

Subproblem 1

$$\max_{\boldsymbol{w}\in\mathcal{W}_R} \quad \min_{\boldsymbol{f}_{\mathcal{N}_k i} \text{ satisfy (2),(3)}} \quad \phi \tag{6}$$

If we solve this Subproblem1 for each $\mathcal{N}_k (k = 1,2,\cdots,C(n, m))$, we can obtain the solution of the original problem (4).

In order to solve Subproblem1, we consider the case where we select not only some $\mathcal{N}_k$ but also some vertex $\boldsymbol{w}_{vj}$ of $\mathcal{W}_R$. Then, we obtain the following subproblem of Subproblem1.

Subproblem 2

$$\begin{array}{ll}\min & \phi \\ \text{subject to} & \sqrt{\boldsymbol{f}_{\mathcal{N}_k i}^T \boldsymbol{f}_{\mathcal{N}_k i}} \leq \phi \ (i = 1, 2, \cdots, m) \\ & \Sigma_{i=1}^{m} \boldsymbol{G}_{\mathcal{N}_k i} \boldsymbol{f}_{\mathcal{N}_k i} = -\boldsymbol{w}_{vj} \\ & \boldsymbol{f}_{\mathcal{N}_k i} \in \mathcal{F}_{f\mathcal{N}_k i} \ (i = 1, 2, \cdots, m)\end{array} \tag{7}$$

Now, we consider the case where we can obtain each optimal solution of Subproblem2 for each $\boldsymbol{w}_{vj} (j = 1, 2, \cdots, l)$ at $\mathcal{N}_k$. Let $\phi_{\mathcal{N}_k, vj}$ be the solution of the Subproblem2 for $\boldsymbol{w}_{vj}$ at $\mathcal{N}_k$. Let $\boldsymbol{f}_{\mathcal{N}_k,j}^* = \left((\boldsymbol{f}_{\mathcal{N}_k 1,j}^*)^T \cdots (\boldsymbol{f}_{\mathcal{N}_k m,j}^*)^T\right)^T$ be the contact forces which give $\phi_{\mathcal{N}_k, vj}$.

Then, the convex polyhedron of the contact forces, whose vertex is $\boldsymbol{f}_{\mathcal{N}_k,j}^*$, can be represented by

$$\begin{aligned}\mathcal{F}_{\mathcal{N}_k} \;\; = \;\; & \{\boldsymbol{f} = \Sigma_{j=1}^{l}\lambda_j \boldsymbol{f}_{\mathcal{N}_k,j}^*, \Sigma_{j=1}^{l}\lambda_j = 1, \\ & \lambda_j \geq 0 (j = 1, 2, \cdots, l)\}\end{aligned} \tag{8}$$

Since the constraints of Subproblem2 is convex, we can resist any $\boldsymbol{w}$ contained in $\mathcal{W}_R$, by using some contact forces contained in $\mathcal{F}_{\mathcal{N}_k}$. Then, the largest $\phi_{\mathcal{N}_k, vj}$ among $\phi_{\mathcal{N}_k, vj} (j = 1, 2, \cdots, l)$ is the optimal solution of Subproblem1 (for $\mathcal{N}_k$). Namely, we can solve Subproblem1 by solving Subproblem2 for each $\boldsymbol{w}_{vj}$.

Based on the formulation by Buss et al. [14], the inequality constraints of Subproblem2 can be rewritten by the following constrains with respect to the symmetric matrices $\boldsymbol{F}_{\mathcal{N}_k i}$ and $\boldsymbol{P}_{\mathcal{N}_k i}$

$$\boldsymbol{F}_{\mathcal{N}_k i} = \begin{pmatrix} \phi \boldsymbol{I} & \boldsymbol{f}_{\mathcal{N}_k i} \\ \boldsymbol{f}_{\mathcal{N}_k i}^T & \phi \end{pmatrix} \succeq \boldsymbol{O}$$

$$\boldsymbol{P}_{\mathcal{N}_k i} = \begin{pmatrix} \mu_{\mathcal{N}_k i} n_{f_{\mathcal{N}_k i}} & 0 & t_{f_{\mathcal{N}_k i},1} \\ 0 & \mu_{\mathcal{N}_k i} n_{f_{\mathcal{N}_k i}} & t_{f_{\mathcal{N}_k i},2} \\ t_{f_{\mathcal{N}_k i},1} & t_{f_{\mathcal{N}_k i},2} & \mu_{\mathcal{N}_k i} n_{f_{\mathcal{N}_k i}} \end{pmatrix} \succeq \boldsymbol{O}$$

where $\boldsymbol{F}_{\mathcal{N}_k i} \succeq \boldsymbol{O}$ means $\boldsymbol{F}_{\mathcal{N}_k i}$ is a positive semidefinite matrix. Then, we can solve Subproblem2 by using positive semidefinite program [15] [16].

Next, we define Relaxed Problem whose constraints are linear and relax the constraints of Subproblem2. Relaxed Problem is a problem to effectively find $\mathcal{N}_k$ for which we don't have to solve Subproblem1. We can use Subproblem2 to find the combinations. But, since simplex method requires less computational time than positive semidefinite program, we use Relaxed Problem. Note that the optimal solution of Relaxed Problem gives a lower bound of the optimal solution of the corresponding Subproblem2, namely of the corresponding Subproblem1, since the constraints of Relaxed Problem contain the constraints of the corresponding Subproblem2. It makes us know $\mathcal{N}_k$ for which we don't have to solve Subproblem1(See next subsection).

Among the constraints of Subproblem2, we approximate $\sqrt{\boldsymbol{f}_{\mathcal{N}_k i}^T \boldsymbol{f}_{\mathcal{N}_k i}} \leq \phi$ by a convex polyhedron circumscribed in the set(Fig.1(a)). We approximate the friction corn(3) by a L-side convex polyhedral corn circumscribed in the friction corn (Fig.1(b)) [17]. Then, we can define Relaxed Problem as follows.

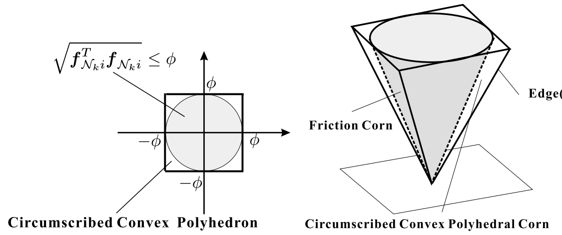
Relaxed Problem

$$\begin{array}{ll}\min & \phi \\ \text{subject to} & \boldsymbol{e}_\kappa^T \boldsymbol{V}_{\mathcal{N}_k i} \boldsymbol{u}_{\mathcal{N}_k i} \leq \phi \\ & (i = 1, 2, \cdots, m)(\kappa = 1, 2, \cdots, d) \\ & -\boldsymbol{e}_\kappa^T \boldsymbol{V}_{\mathcal{N}_k i} \boldsymbol{u}_{\mathcal{N}_k i} \leq \phi \\ & (i = 1, 2, \cdots, m)(\kappa = 1, 2, \cdots, d) \\ & \Sigma_{i=1}^{m} \boldsymbol{G}_{\mathcal{N}_k i} \boldsymbol{V}_{\mathcal{N}_k i} \boldsymbol{u}_{\mathcal{N}_k i} = -\boldsymbol{w}_{vj}\end{array} \tag{9}$$

where $\boldsymbol{V}_{\mathcal{N}_k i} \in \mathcal{R}^{d \times L}$ denotes the matrix whose column is a unit edge vector of the frictional convex polyhedral cone $\boldsymbol{v}_{\mathcal{N}_k i_j}$ $(\boldsymbol{V}_{\mathcal{N}_k i} = (\boldsymbol{v}_{\mathcal{N}_k i_1} \cdots \boldsymbol{v}_{\mathcal{N}_k i_m}))$, $\boldsymbol{u}_{\mathcal{N}_k i} (\geq \boldsymbol{o}) \in \mathcal{R}^L$ denotes the vector whose jth element represents the magnitude of the contact force in $\boldsymbol{v}_{\mathcal{N}_k i_j}$ direction, and $\boldsymbol{e}_\kappa \in \mathcal{R}^d$ denotes the vector whose κth element is 1 and whose other elements are 0(for example, $\boldsymbol{e}_1 = (1\ 0\ 0)^T$ in 3 dimensional space). Note that we can solve Relaxed Problem by simplex method.

3.2 Procedure of the Algorithm

In this subsection, we describe the algorithm to search the optimal $\mathcal{N}_k$. Now, let $\hat{\rho}$ be the temporary optimal solution, $\rho_{\mathcal{N}_k}$ be the optimal solution of



(a) $\sqrt{\boldsymbol{f}^T_{\mathcal{N}_k i}\boldsymbol{f}_{\mathcal{N}_k i}} \leq \phi$ (b) Friction Corn

Figure 1: Convex Polyhedron and Convex Polyhedral Corn Circumscribing the Constraints of Subproblem2

Subproblem1 for $\mathcal{N}_k$, and $\hat{\rho}_{\mathcal{N}_k}$ be its temporary optimal solution. We represent a list of feasible $\mathcal{N}_k$ as LIST.

step 1 We put all candidate $\mathcal{N}_k$ into LIST. Let $\hat{\rho}$ be an appropriate lower bound value. Let each $\rho_{\mathcal{N}_k}$ and each $\hat{\rho}_{\mathcal{N}_k}$ for each $\mathcal{N}_k$ be appropriate upper and lower bound values respectively.

step 2 We solve Subproblem1 for some $\mathcal{N}_k$ contained in LIST.

step 3 If we can get the solution of the Subproblem1 in step2, let $\hat{\rho}$ $(=\rho_{\mathcal{N}_k} = \hat{\rho}_{\mathcal{N}_k})$ be the solution and $\hat{\boldsymbol{w}}_{vj}$ be $\boldsymbol{w}_{vj}$ which gives the solution. Otherwise, we eliminate the $\mathcal{N}_k$ from LIST and go back to step2.

step 4 We solve Relaxed Problem for $\hat{\boldsymbol{w}}_{vj}$, at each $\mathcal{N}_k$ contained in LIST. If we cannot get the solution at some $\mathcal{N}_k$, we eliminate this $\mathcal{N}_k$ from LIST. If we can get the solution $\hat{\rho}_{\mathcal{N}_k, \hat{v}j}$ at some $\mathcal{N}_k$, we compute $\hat{\rho}_{\mathcal{N}_k}$ $= \max\{\hat{\rho}_{\mathcal{N}_k},\ \hat{\rho}_{\mathcal{N}_k, \hat{v}j}\}$. If $\hat{\rho} < \hat{\rho}_{\mathcal{N}_k}$, we eliminate this $\mathcal{N}_k$ from LIST.

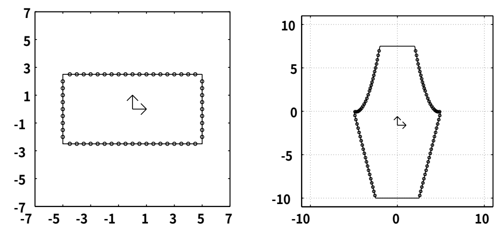
step 5 Let $\hat{\mathcal{N}_k}$ be $\mathcal{N}_k$ at which $\hat{\rho}_{\mathcal{N}_k, \hat{v}j}$ is the least among those at all $\mathcal{N}_k$ contained in LIST.

step 6 We solve Subproblem1 for $\hat{\mathcal{N}_k}$. If we can get the solution, let $\rho_{\mathcal{N}_k}$ $(= \hat{\rho}_{\mathcal{N}_k})$ be the solution and $\hat{\boldsymbol{w}}_{vj}$ be $\boldsymbol{w}_{vj}$ which gives the solution. If $\hat{\rho} > \rho_{\hat{\mathcal{N}_k}}$, $\hat{\rho} = \rho_{\hat{\mathcal{N}_k}}$. If we cannot get the solution or $\hat{\rho} < \rho_{\hat{\mathcal{N}_k}}$, we eliminate this $\mathcal{N}_k$ from LIST and go back to step5.

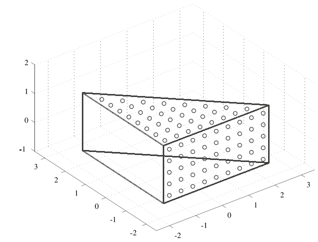
step 7 If we can get the relation $|\hat{\rho} - \rho_{\mathcal{N}_k}| < \epsilon$ (ϵ denotes an arbitrary small positive value) for all $\mathcal{N}_k$ contained in LIST, we finish the loop. Otherwise, we go back to step4.

4 Numerical Examples

In order to show the effectiveness of our approach, we show some numerical examples in this section. We show the target objects and the candidate contact points in Fig.2. The objects in Case I and II are in 2 dimensional space. The object in Case III is a triangular prism whose base is a right isosceles triangle($4 \times 4 \times 4\sqrt{2}$). In this figure, the points on each object indicate the candidate contact points. Note that

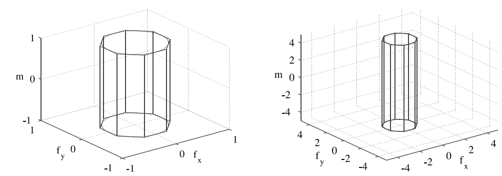


(a) Case I (b) Case II



(c) Case III

Figure 2: Target Objects and Candidate Contact Points



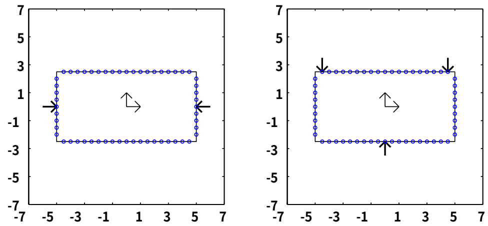
(a) Case I (b) Case II

Figure 3: Required External Force Sets in 2 dimensional space

we show only the candidate contact points on the top face and on one side in Case III for easy to see. We give the candidate contact points on the bottom face and on the other side in the same way as those on the top face and on the side respectively. The number of the candidate contact points is 56 in Case I, 86 in Case II, and 217 in Case III (36 for top and bottom face, 45, 45, and 55 for 3 sides). Let $\Sigma_{O_i}(i=$I,II,III$)$ be the object coordinate frame fixed at each geometric center of the object in Case i. We use a frictional 16-side convex polyhedral corn in Relaxed Problem. We set $\mathcal{W}_R$ as follows:

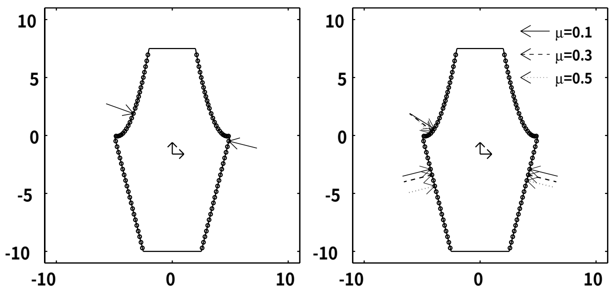
$$\mathcal{W}_R = \begin{cases} \{(\boldsymbol{f}^T,\ \boldsymbol{m}^T)^T \,||\boldsymbol{f}| \leq sg, \\ -0.8\gamma sg \leq m_1 \leq 0.8\gamma sg\} \quad \text{(in Case I and II)} \\ \{(\boldsymbol{f}^T,\ \boldsymbol{m}^T)^T | -sg \leq f_i \leq sg, \\ -0.8\gamma sg \leq m_i \leq 0.8\gamma sg \\ (i=1,2,3)\} \quad \text{(in Case III)} \end{cases}$$

where $\boldsymbol{f}$ and $\boldsymbol{m}$ denote the external force and moment respectively, f_i and m_i denote the ith component of $\boldsymbol{f}$ and $\boldsymbol{m}$ respectively, s denotes the area or volume of each object, and g=0.01 denotes the specific gravity of



(a) 2 Fingers (b) 3 Fingers

Figure 4: Optimal Grasp Points in Case I



(a) 2 Fingers (b) 3 Fingers

Figure 5: Optimal Grasp Points in Case II

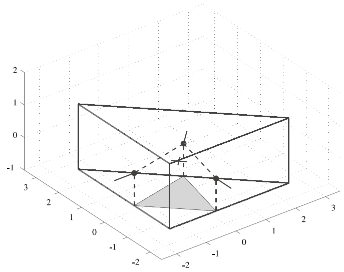


Figure 6: Optimal Grasp Points in Case III

each object. γ in Case i (i =I,II) denotes the distance between Σ_{O_i} and the vertex which is closer to Σ_{O_i} than any other vertexes. γ in Case III denotes the distance between $\Sigma_{O_{III}}$ and the face which is closer to $\Sigma_{O_{III}}$ than any other faces. Here we suppose the case where gravitational force (external force) can be applied to each object in arbitrary direction and where the external moment can be applied to each object resulting from that the position of the center of gravity can be located in the circle (Case I and II) or the sphere (Case III), whose center is the origin of Σ_{O_i} and whose radius is 0.8γ. $\mathcal{W}_R$ in Case I and II are convex sets but not convex polyhedrons. Then, we approximate the sets by 10-side convex polyhedrons circumscribed in the sets (Fig.3).

In Case I and II, we compute in the cases where the number of the fingers is 2 and 3 and where the

Table 1: ρ in the Problem (4)

(a) Case I

μ	0.1	0.3	0.5
2 fingers	3.71	1.38	0.919
3 fingers	3.13	0.978	0.724

(b) Case II

μ	0.1	0.3	0.5
2 fingers	35.1	6.44	3.92
3 fingers	6.21	2.77	2.01

frictional coefficient is 0.1, 0.3, and 0.5. In Case III, we set the number of the fingers is 3 and the frictional coefficient is 0.3. Note that if we obtain some optimal combinations of contact points excepting symmetric arrangements, we search the optimal combination which minimizes the criterion $\max_{\boldsymbol{w}\in\mathcal{W}_R} \min_{\boldsymbol{f}_{\mathcal{N}i} \text{ satisfy } (2),(3)} \sqrt{\Sigma_{i=1}^m \boldsymbol{f}_{\mathcal{N}i}^T \boldsymbol{f}_{\mathcal{N}i}}$ among these obtained combinations.

Results about the optimal combination of contact points are shown in Fig.4, Fig.5, and Fig.6. Results about ρ in the problem (4) in Case I and II are shown in Table.1. ρ in Case III was 0.669. In Case III, the obtained optimal grasp points were $(2/9, -4/3, 0)^T$, $(2/3, 2/3, 0)^T$, and $(-4/3, 2/9, 0)^T$ (the origin (X) is the origin of $\Sigma_{O_{III}}$). In Fig.4 and Fig.5, (a) and (b) show the results in the cases where the object is grasped by 2 and 3 fingers respectively, and the arrows show the obtained optimal grasp points. Note that we show only one optimal combination of contact points where the arrangements of the obtained 2 optimal combinations are symmetric. Note also that we show only one optimal combination of contact points in Fig.4 and Fig.5, excepting the case shown in Fig.5(b), since the optimal grasp points didn't depend on the magnitude of the frictional coefficient. In Fig.6, the arrows, whose tip is black sphere, show the obtained optimal grasp points, X shows the origin of the reference frame. The gray triangle shows the projection drawing of the triangle, made by the 3 optimal grasp points, on the bottom face of the object.

From Fig.6, we can see that the each optimal grasp point is located at a position close to each foot of each perpendicular from $\Sigma_{O_{III}}$ to each side. For someone's information, we compute ρ in the case where we grasp the object at these feet. ρ was 0.718 (which is bigger than 0.669). From Table.1, we can see that the necessary magnitudes of the contact forces to resist the required external force depends both on the magnitude of the frictional coefficient and on the number of the fingers in Case I and II.

The above algorithm was implemented in C/C++ and the above calculations were done on a PC with 1.2GHZ ATHLON. In 2 dimensional space (the fictional coefficient is 0.3), the running times for Case I and II, respectively, were about 5 and 7 seconds in the case where we grasp by 2 fingers, and about 8 and 3 minutes in the case where we grasp by 3 fingers. In 3 dimensional space, the running time was about 25 minutes. For someone's information, we computed in the case where we use Subproblem2 in place of Relaxed Problem to effectively find $\mathcal{N}_k$ for which we don't have to solve Subproblem1 (the fictional coefficient is 0.3). In this case, the running times in the cases where we grasp by 2 and 3 fingers, respectively, were about 24 seconds and 1.2 hours in Case I, and about 41 seconds and 8.7 hours in Case II. The running time was about 4.35 hours in Case III. We think this shows the effectiveness of using Relaxed Problem. The numbers of $\mathcal{N}_k$ at which we solved Subproblem2 for all vertexes of $\mathcal{W}_R$ in the cases where we grasp by 2 and 3 fingers, respectively, were 6 and 278 in Case I (the fictional coefficient is 0.3), and 9 and 53 in Case II (the fictional coefficient is 0.3). In Case III, the number was 21. Note that the other $\mathcal{N}_k$ were eliminated. We think this shows the effectiveness of the branch-and-bound method.

In our approach, the number of the candidate combinations of contact points is $C(n, m)$ and then the number become large when the number of the fingers or the candidate contact points is large. If the running time is too large to apply the above algorithm, we can get approximate optimal grasp points in a smaller running time by the following way; at first we reduce the candidate contact points and search optimal grasp points, and then we search optimal grasp points from original candidate contact points which is close to the obtained optimal grasp points.

5 Conclusion

In this paper, we have searched optimal grasp points on a grasped object by using the concept of required external force set. By using the required external force set, we can deal with general desired grasps including force closure and equilibrium grasp, and can evaluate the magnitudes of the resistible external forces and moments. We have defined an optimal grasp that minimizes the magnitude of the contact force required to resist any external force and moment contained in the given required external force set. We have searched the optimal grasp points by using a branch-and-bound method. We have also presented some numerical examples in order to show the validity of our approach.

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