Four components three dimensional FEM analysis of flux concentration apparatus with four plate

FOUR COMPONENT THREE DIMENSIONAL FEM ANALYSIS OF FLUX CONCENTRATION APPARATUS WITH FOUR PLATES

Takeshi Yoshimoto , Sotoshi Yamada and Kazuo Bessho

ABSTRACT

This paper deals with the four component three dimensional FEM analysis of a flux concentration model with four thick conducting plates placed between a pair of a.c.-excited coils. We have already analyzed a flux concentration apparatus with two conducting plates by using a newly developed iterative 3-D calculation method.[1] [2] An improved formulation is also treated in this paper. This calculation method is applied to the present four-plate type model, to prove its adaptability and to search for more efficient design of such apparatus. Distributions of flux densities, eddy currents and scalar potentials are calculated and discussed.

The role of the scalar potential for the 3-D eddy current diffusion problems is also interpreted, based on our calculation results.

INTRODUCTION

For the eddy current diffusion problems, the real three dimensional or axi-symmetrical analysis method is desirable, because eddy currents essentially circulate in a finite length conductor three dimensionally.

A flux concentration apparatus which has been investigated at Kanazawa University since 1981 [3], requires a full three dimensional analysis to clarify the effect of eddy currents.

Four component three dimensional FEM formulations, shown in references [4] or [5], intrinsically lead to large and sparse system matrices due to four components per node in addition to the 3-D structure. Our calculation method is devised to avoid using large computer memories by dividing the total simultaneous equations into four groups and using an iterative method. Gauss elimination method is used in solving each subsystem of equations.

As for the controversial theme of the role of the scalar potential [6] [7], one inductive conclusion is led, on the basis of the real calculated distribution of the scalar potential in our model. This distribution is also made possible to obtain by the ability of our calculation method to treat large models. We make it clear that the scalar potential plays a great role in obtaining circulating eddy currents in a finite shaped conductor, especially when it has some discontinuity in the stream-lines similar to the flow of the impressed currents.

FIELD ANALYSIS EQUATIONS

For a three dimensional quasi-stationary eddy current diffusion problem, we obtain first the expression of the eddy current density, from combining Maxwell's electromagnetic field equations.

$$\overline{J}_e = -j\omega\sigma\overline{A} - \sigma\,\mathrm{grad}\,\phi \qquad (1)$$

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Next, we get the following fundamental equation, assuming constant μ in the x, y, and z directions.

$$\frac{1}{\mu}\nabla(\nabla\overline{A}) - \frac{1}{\mu}\nabla^2\overline{A} = -\sigma(j\omega\overline{A} + \nabla\phi) + \overline{J}_s \qquad (2)$$

The other equation is derived from the fact that the divergence of the eddy current density should equal zero.

$$\mathrm{div}\left\{\sigma(j\omega\overline{A} + \mathrm{grad}\ \phi)\right\} = 0 \qquad (3)$$

where,

$\overline{A}$: three component vector potential
$\overline{J}_s$: source current density
ϕ : electric scalar potential
μ : permeability of medium
σ : conductivity of medium

The above two equations, (2) and (3) are the fundamental field analysis equations necessary to determine the vector potential and the scalar potential distributions for eddy current problems.

Now we introduce, from the standpoint of the computer memory, one iterative method to avoid treating the whole system of equations. From (3), the vector potential $\overline{A}$ can be written as

$$\nabla\overline{A} = -\nabla^2\phi/j\omega \qquad (4)$$

Using (4), (2) can be decomposed into three axis-component equations as shown in (5)-(7). Adding (3), the following four equations are obtained.

$$-\frac{1}{\mu}\nabla^2 A_x + j\omega\sigma A_x = J_{sx} - (\sigma\nabla\phi)_x + \frac{1}{j\omega\mu}\frac{\partial}{\partial x}(\nabla^2\phi) \qquad (5)$$

$$-\frac{1}{\mu}\nabla^2 A_y + j\omega\sigma A_y = J_{sy} - (\sigma\nabla\phi)_y + \frac{1}{j\omega\mu}\frac{\partial}{\partial y}(\nabla^2\phi) \qquad (6)$$

$$-\frac{1}{\mu}\nabla^2 A_z + j\omega\sigma A_z = J_{sz} - (\sigma\nabla\phi)_z + \frac{1}{j\omega\mu}\frac{\partial}{\partial z}(\nabla^2\phi) \qquad (7)$$

$$\nabla\cdot(\sigma j\omega(A_x\overline{u}_x + A_y\overline{u}_y + A_z\overline{u}_z) + \sigma\nabla\phi) = 0 \qquad (8)$$

FORMULATION BY GALERKIN METHOD

Formulation is made by using Galerkin method with shape functions N_i as weighting functions. Supposing (5) as a representative of (5)-(7), we show the transformation of (5) only. Its integral form is

$$-\int_V\left\{N_i\frac{1}{\mu}\nabla^2 A_x\right\}dv + j\omega\sigma\int_V N_i A_x\cdot dv = \int_V N_i J_{sx}\cdot dv - \int_V N_i(\sigma\nabla\phi)_x\cdot dv + \frac{1}{j\omega\mu}\int_V N_i\cdot\frac{\partial}{\partial x}(\nabla^2\phi)\cdot dv \qquad (9)$$

Using the divergence theorem and the vector identity

$$\nabla(N_i\overline{R}) = N_i(\nabla\overline{R}) + (\nabla N_i)\overline{R} \qquad (10)$$

The first term on the left side of (9) is transformed to (11).

$$-\int_V\left\{N_i\frac{1}{\mu}\nabla^2 A_x\right\}dv = -\frac{1}{\mu}\int_S N_i\frac{\partial A_x}{\partial n}dS$$

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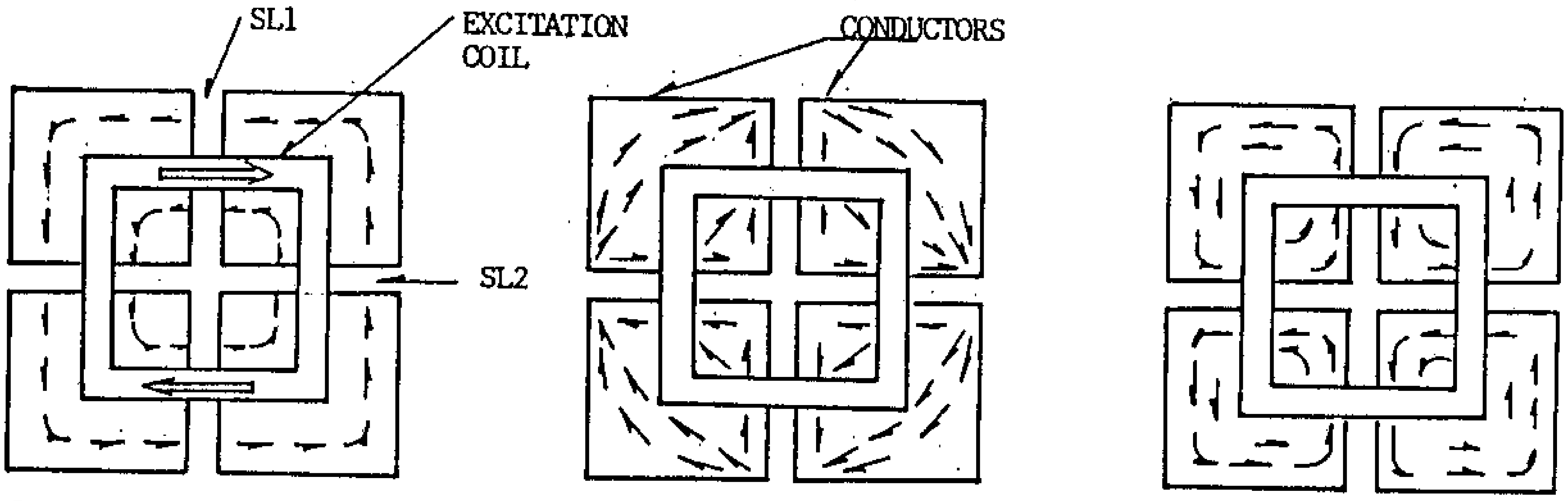

(a) Eddy Currents by Vector Potential (b) Eddy Currents by Scalar Potential (c) Synthesis of the Two

Fig.8 Composition of Eddy Currents

into consideration when we treat a model including conductors with any discontinuity or any gaps in the stream-lines similar to the flow of the impressed currents. Though there are several papers stating that the scalar potential results from or related to the impressed voltage [7], our conclusion obtained from above calculation is that the scalar potential provides a voltage source for eddy currents to be continuous in a finite shaped conductor, which signifies that it compensates the incomplete expression of the eddy current density by the vector potential $\overline{A}$ only.

CONCLUSION

A flux concentration apparatus with four thick conducting plates is analyzed by using our newly developed 3-D iterative calculation method. Two slit-widths of SL1 and SL2 are confirmed to affect the concentration of the magnetic flux in the central part of the air-gap. It is also confirmed that the flux concentration is due to the intensified eddy currents in the edges adjacent to the air-slit.

The role of the scalar potential in the 3-D analysis is clarified on the basis of the real calculated results. The scalar potential is concluded to be necessary when we treat such a model including conductors with any discontinuity or any gaps in the stream-lines similar to the flow of the impressed currents.

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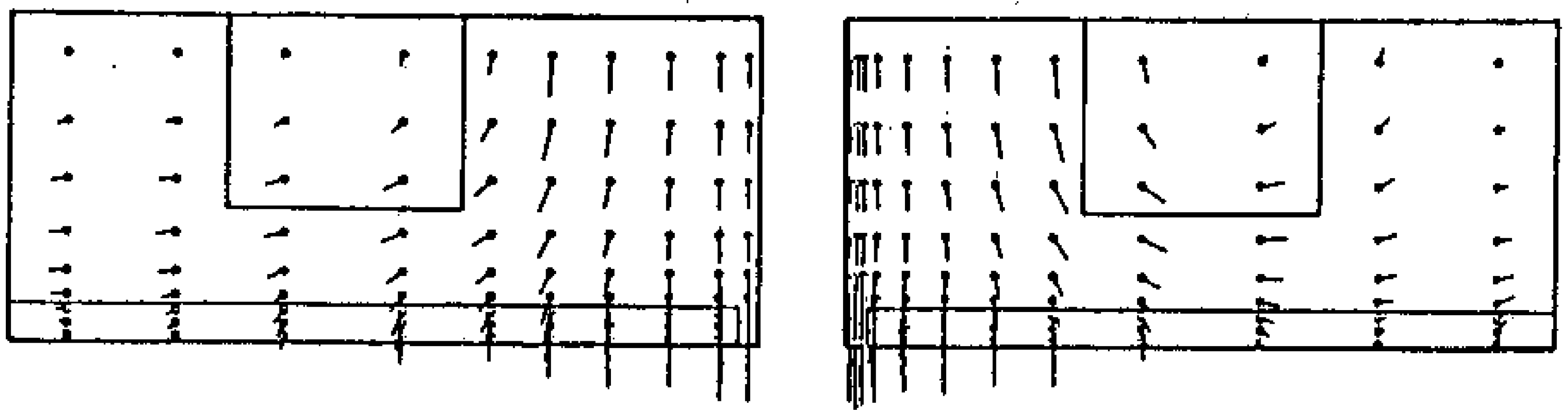
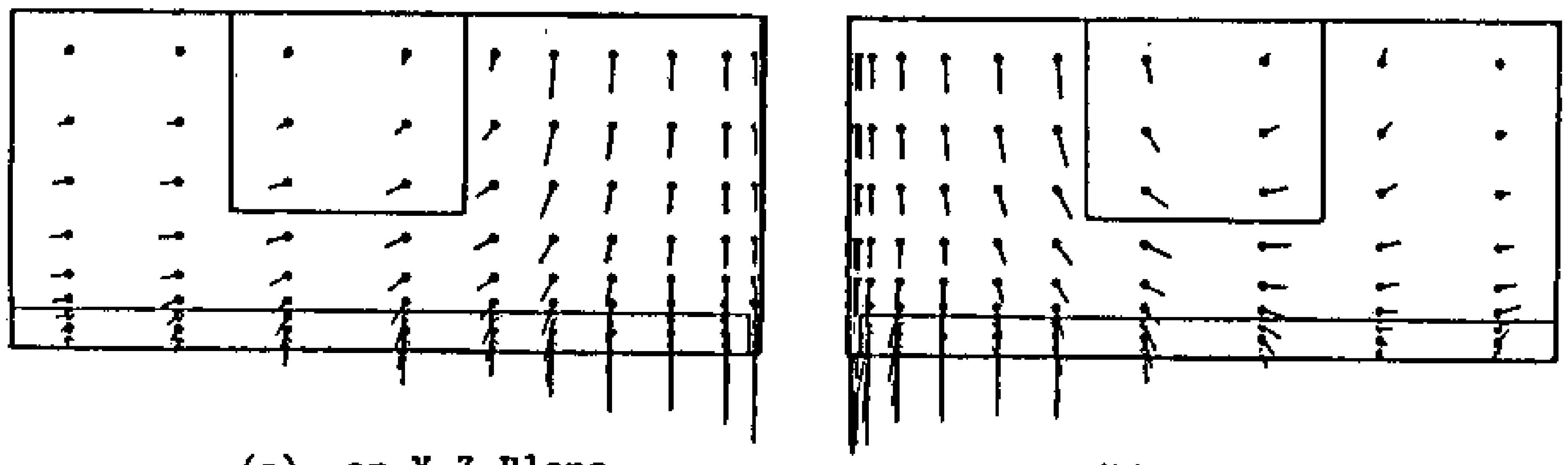


(a) on Y-Z Plane (b) on X-Z Plane

Fig.4 Flux Distribution (SL1=10 mm, SL2=10 mm)



(a) on Y-Z Plane (b) on X-Z Plane

Fig.5 Flux Distribution (SL1= 6 mm, SL2= 5 mm)

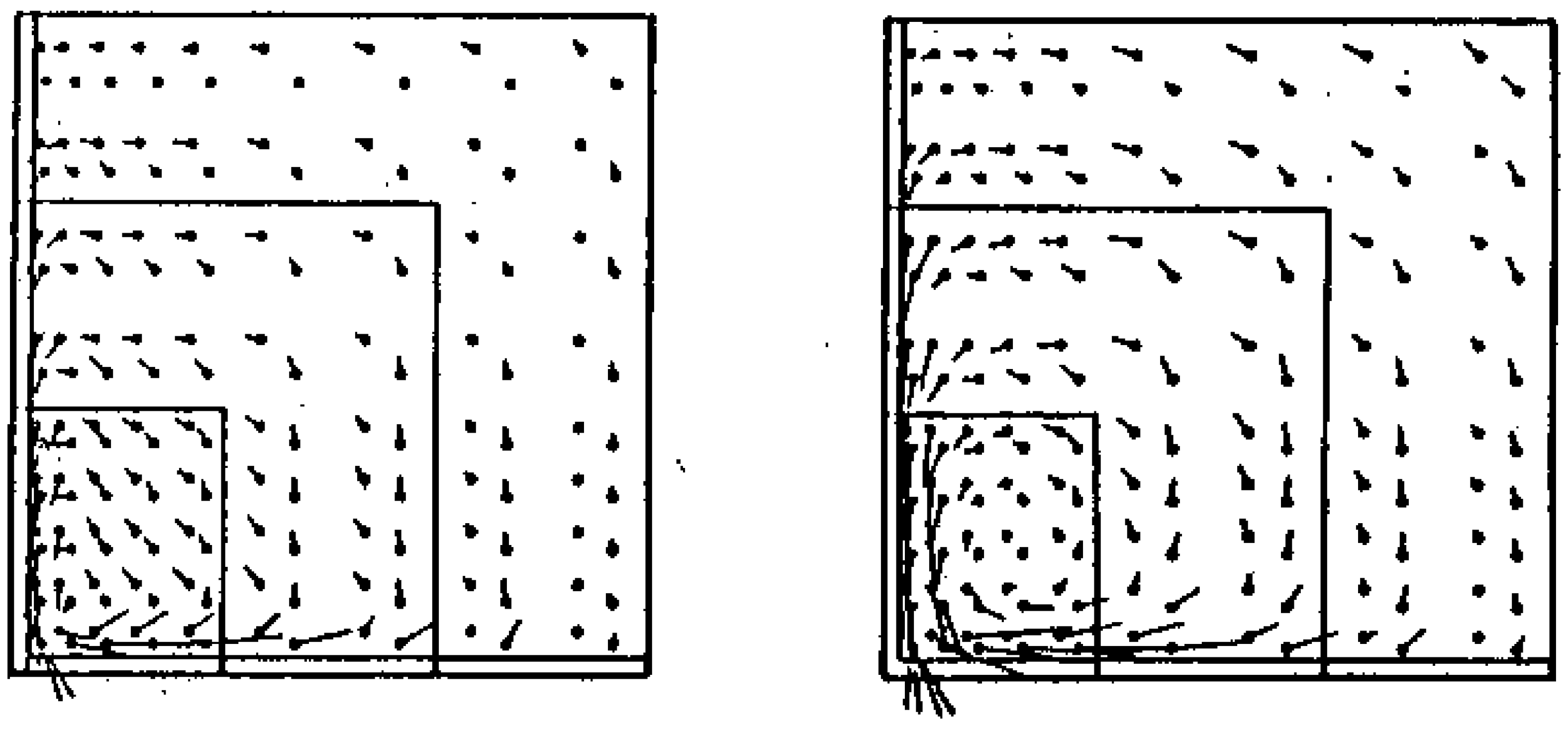


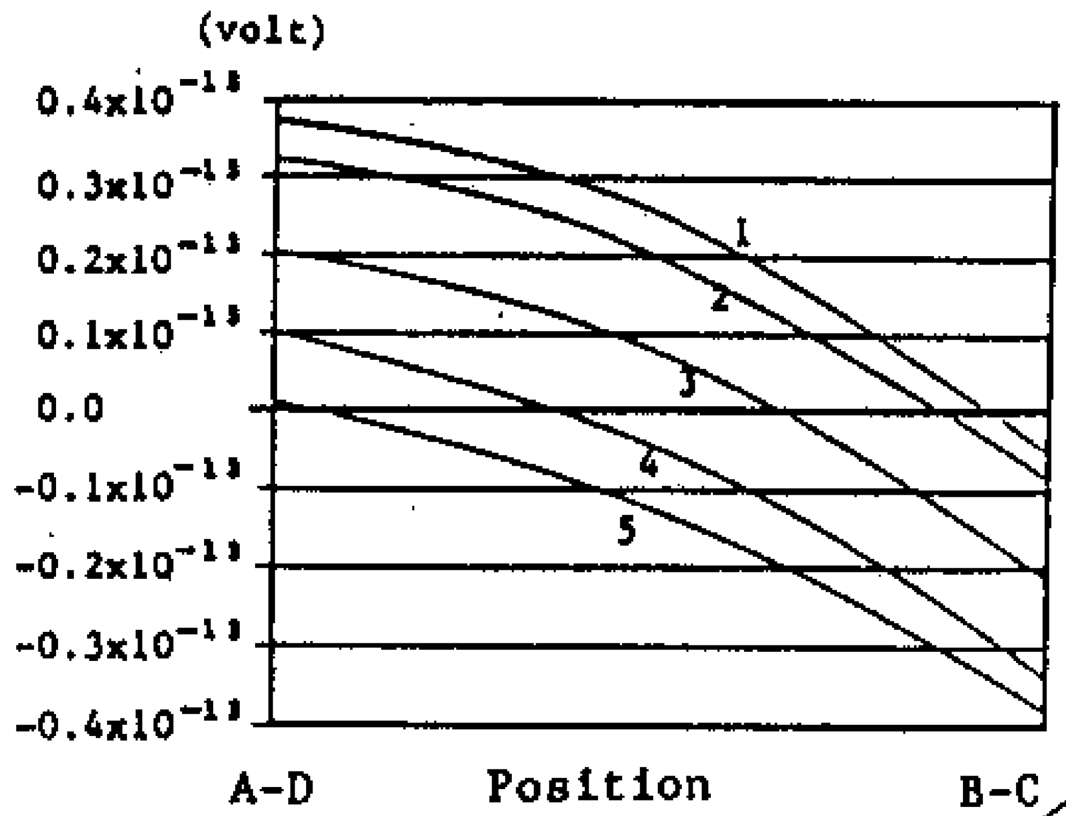
Fig.6 Eddy Current Distribution (SL1=10 mm, SL2=10 mm)

(a) on Central Plane

(b) on Top Surface

Fig.6 shows the eddy current distributions on the central plane and the top surface of a conductor plate for SL1=10 mm and SL2=10 mm. It is observed that eddy currents concentrate near the central part and contribute to strengthen z-ward flux density.

Distributions of the scalar potential on the top surface of the right-hand and backward plate are shown in Fig.7. The gradient of the scalar potential constitutes a part of eddy currents together with the differential of the vector potential. It is recognized that the eddy currents calculated from Fig.7 contribute to make eddy currents circulate in each conductor.



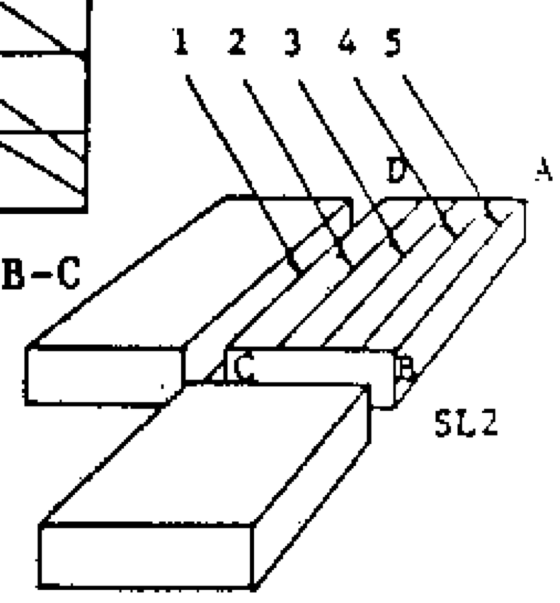


Fig.7 Scalar Potential Distribution on Top Surface of a Conductor

THE EFFECT OF THE SCALAR POTENTIAL ON EDDY CURRENTS

To study the role of the scalar potential, the expression of eddy current density is restated as

$$\overline{J}_e = -j\omega\sigma\overline{A} - \sigma \,\mathrm{grad}\; \phi \qquad (1)$$

The first term indicates the component by the vector potential, while the second term is the component by the gradient of the scalar potential. If we don't take ϕ into account, the calculation results show that eddy currents flow in opposite direction to the route of the impressed currents. Because the direction of the vector potential $\overline{A}$ is decided mainly by the impressed currents. As a result, there exist eddy currents perpendicular to the boudaries, B-C and C-D, which is physically impossible. In the next place, when we consider the scalar potential, the results show that the second component of the eddy current cancels the flow perpendicular to the boundaries of B-C and C-D, so that eddy currents turn to circulate in the conductor.

The synthesis of the eddy currents, based upon the idea mentioned above, is illustrated in Fig.8 (a), (b) and (c). Figure (a) shows the eddy currents decided by the vector potential only, while figure (b) shows the eddy currents by the gradient of the scalar potential. Figure (c) shows the synthesized ones.

The inference from the above calculated results of our model, shows the scalar potential must be taken

$$-\frac{1}{\mu}\int_V\left(\frac{\partial N_i}{\partial x}\frac{\partial A_x}{\partial x}+\frac{\partial N_i}{\partial y}\frac{\partial A_x}{\partial y}+\frac{\partial N_i}{\partial z}\frac{\partial A_x}{\partial z}\right)dv \qquad (11)$$

The first term on the right side of (11) vanishes in the case Neumann condition holds on the surface enclosing the region. In addition, it should be said the third term on the right side of (9) is significant only in the case that the scalar potential is expressed in an equation of order less than 3. If it is expressed in an equation of order less than 2, the field considered becomes equal to one where the Coulomb gauge holds. When we adopt a brick element of order less than 2, (9) reduces to (12), with the same transformation available for (6) and (7).

$$-\frac{1}{\mu}\int_V(\nabla N_i)(\nabla A_x)\,dv + j\omega\sigma\int_V N_i A_x\,dv = \int_V N_i J_{sx}\,dv - \int_V N_i(\sigma\nabla\phi)_x\,dv \qquad (12)$$

As for (8), using shape functions N_i as weighting functions and applying the divergence theorem besides the vector identity (10), the following equation is obtained:

$$\int_S N_i\sigma(j\omega\bar{A}+\nabla\phi)\cdot\bar{n}\,dS - \int_V(\nabla N_i)\sigma(j\omega\bar{A}+\nabla\phi)\,dv = 0 \qquad (13)$$

Considering that the total outflow from the region equals zero, (13) become

$$\int_V(\nabla N_i)j\omega(A_x\bar{u}_x+A_y\bar{u}_y+A_z\bar{u}_z)\,dv + \int_V(\nabla N_i)\nabla\phi\,dv = 0 \qquad (14)$$

Therefore, (12) and (14) will be the two fundamental equations to accomplish the finite element formulation for 3 dimensional eddy current diffusion problems, whatever finite elements are used.

ITERATIVE CALCULATION METHOD

The global system matrix obtained by discretization of (5)-(8), will be generally not only large but also sparse due to the introduction of the scalar potential. By moving ϕ onto the right-hand side as shown in (5)-(7) and assuming its value constant in each calculation of (5)-(7), variables, A_x, A_y and A_z, can be solved independently. This means we have only to treat one sub-system of equations at a time. These variables decide ϕ by using (8). The new ϕ is put back again into (5)-(7) to calculate new variables of A_x, A_y and A_z. In this iteration process, final converged values of A_x, A_y, A_z and ϕ can be obtained. Under-relaxation method is used for updating the scalar potential. The method we propose is based upon the idea that the four groups of equations could be regarded as respective sub-systems of equations regarding A_x, A_y, A_z and ϕ, as shown in Fig.1. Though iteration process is needed among the four groups, the number of elements of each bandmatrix decreases to one sixteenth of that of the global system matrix. This memory saving of 1/16 is very effective in the circumstance that the general time sharing system of the large-sized computer provids a certain limited region to an individual user. In addition, it also enables to use the direct calculation method like Gauss elimination.

FOUR-PLATE TYPE MODEL

A basic model of our flux concentration apparatus with four conducting plates is shown in Fig.2, while its configuration is shown in Fig.3. A pair of two exciting coils induce eddy currents in the four conducting plates. The flux density in the central part of the air-gap is intensified when two slit-widths, SL1 and SL2, among the four plates become small.

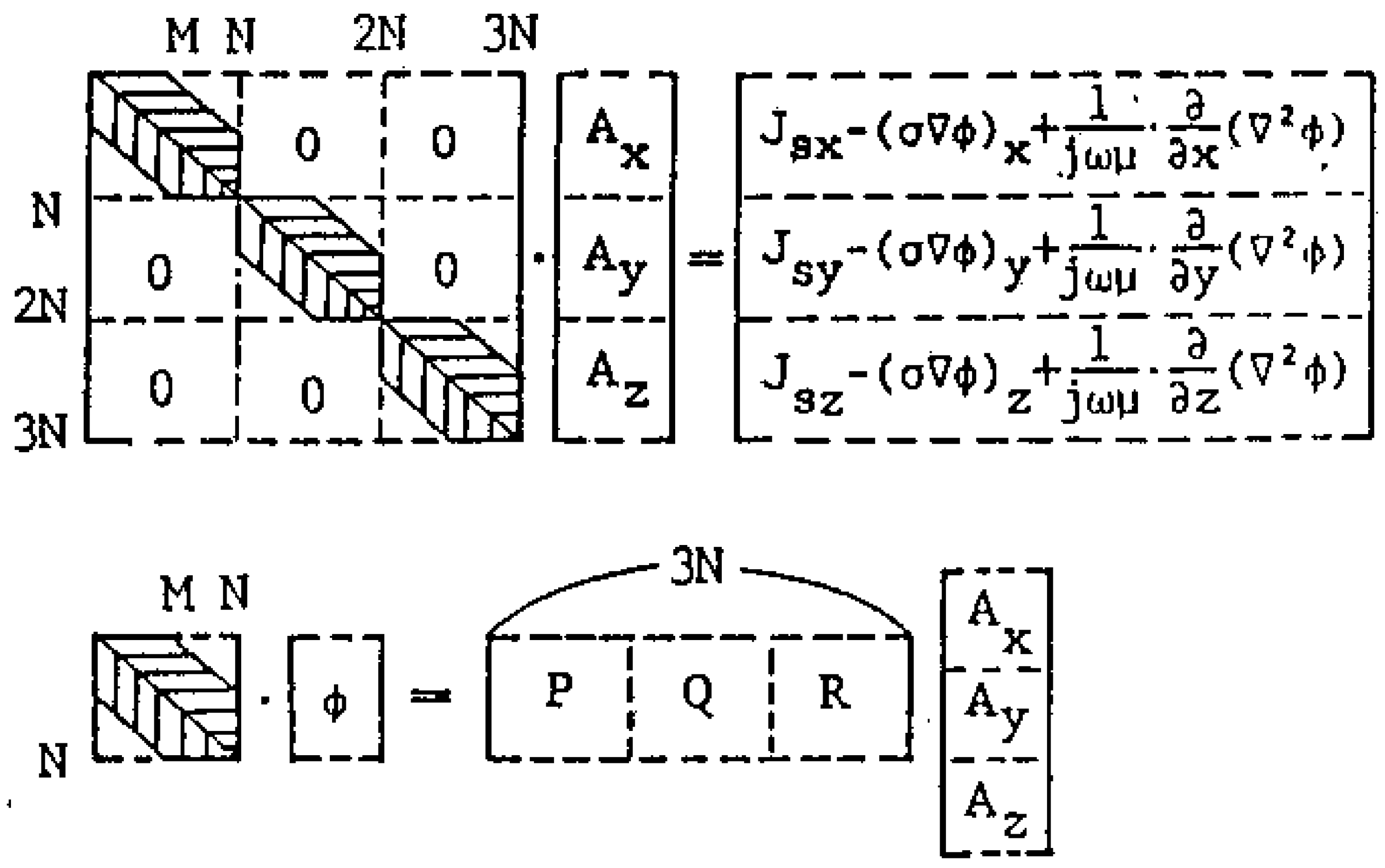


Fig.1 Matrix Structure (N:Number of total nodes, M:Bandwidth in the case of one variable)

To analyze this 3-D model, the four component iterative method mentioned above is used. Using 3-D symmetry, one eighth of the whole region is discretized and calculated. As the model is layer type, the region is divided by using first-order triangular prisms. Neumann condition is imposed on the outer boundary surface, with zero vector potential along z-axis.

Frequency of the impressed currents is 60 Hz, the conductivity of the copper plate 8.62×10^7 S/m, and the ampere turns of the excitation coil are 2.0×10^{-1} ^{2}AT/mm^2. Distributions of the flux density, the eddy current density and the scalar potential are calculated.

Fig.4 shows the flux density distribution for SL1=10 mm and SL2=10 mm, while Fig.5 shows another one for SL1=6 mm and SL2=5 mm. In both figures, (a) and (b) show the flux distributions for Y-Z plane and X-Z plane, respectively.

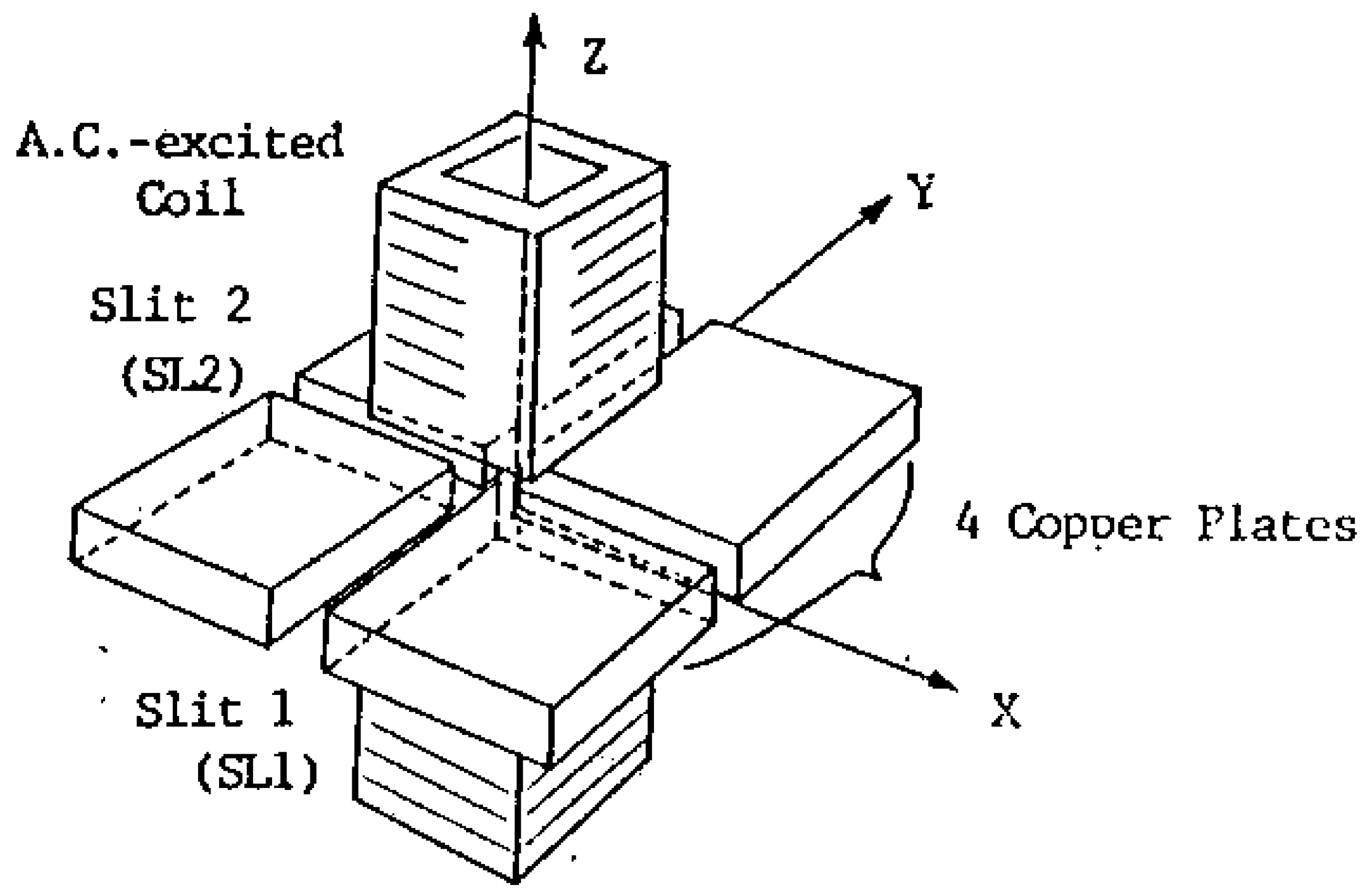


Fig.2 Flux Concentration Model with Four Conducting Plates

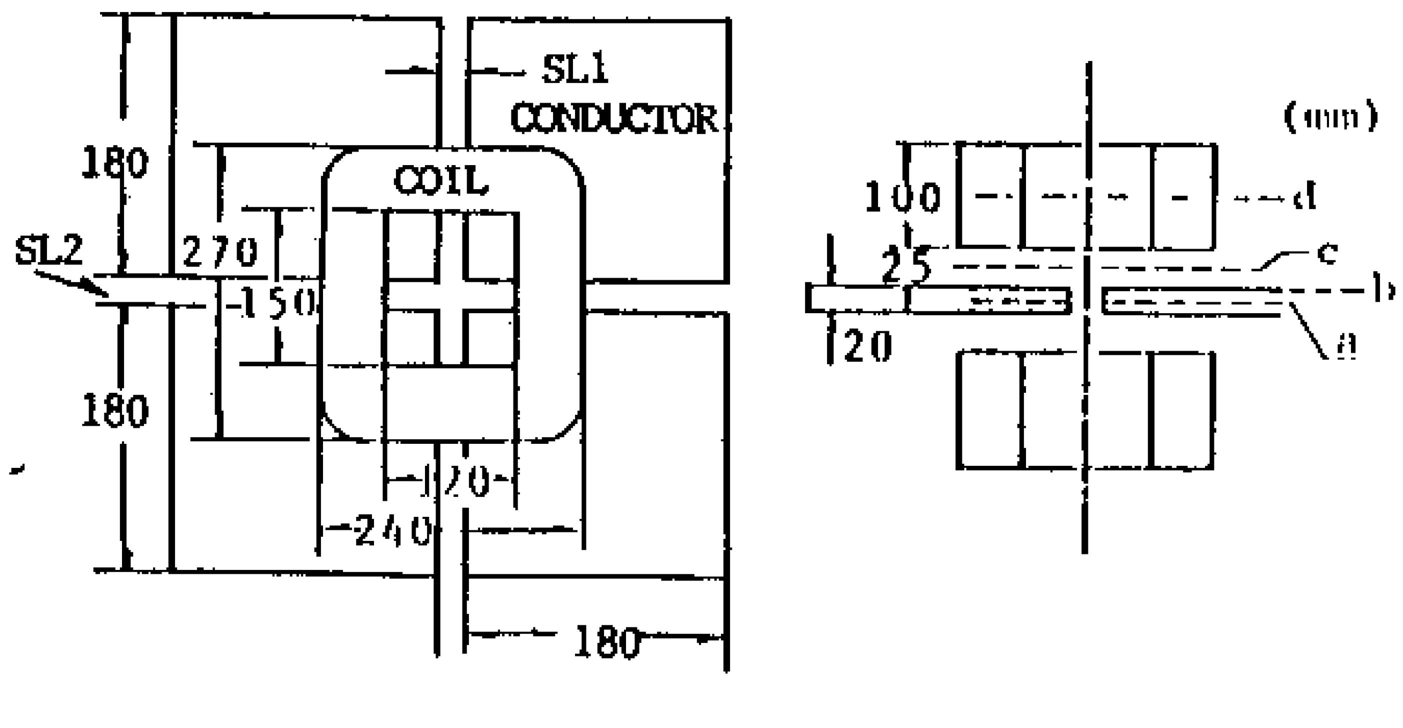


Fig.3 Model Configuration