## Paper

# Accuracy Evaluation on Area Measurement using Pseudorandom Pixel Placement for Low Resolution Images 

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#### Abstract

Conventional imaging systems have the pixels that are arranged in the regular lattice positions, or lattice pixel placement (LPP). LPP is employed in most imaging systems for its advantages on pixel read-out and image reconstruction ways, however, in LPP, the clarity on image representation depends on the direction of the object in the image, or the directional dependency exists. In this paper, we propose the pseudorandom pixel placement (PPP) for reducing the directional dependency on the accuracy in the area measurement. We carried out the simulation to evaluate the directional dependency decrease effect for various pixel parameters, and discuss the optimum pixel parameters and the image resolution.


Key words: area measurement, pseudorandom pixel placement, directional dependency, pixel parameter, image resolution

## 1. Introduction

Conventional imaging systems have the pixels arranged in the regular lattice positions, or lattice pixel placement (LPP). LPP has a large advantages on pixel read-out and image reconstruction methodologies based on raster scan procedure, and is used in most imaging systems. However, in the image capture and representation using LPP process, the number of the pixels contained in a solid object in the image depends on the direction of the object, as shown in Fig.1, where the dark gray represents the target object, and the light gray pixels represent the pixels composing the object. In the image representation using LPP, the jaggy also appears at the slant line edge of the object. As shown above, the quality of the image representation using LPP depends on the direction of the object in the image, or the "directional dependency" we call in this paper, and it results in the degraded accuracy in the image instrumentation, such as the area measurement, especially with low resolution.

The authors have been proposing the pixel placement strategy for reducing the directional dependency effect with the practical implementation, "pseudorandom pixel placement $(\mathrm{PPP})^{1)}{ }^{3)}$ ". The PPP is based on the idea to randomly displace the representing point in the pixel, the photo detector or the light emitter

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Fig. 1 Example of the directional dependency.
(we call them "active area" in this paper) to form the whole image from the regular lattice point. The image representation using PPP has advantages on reducing the directional dependency against that using LPP. The authors' previous interests are focused on the image clearness in terms of how the users perceive, and CMOS image sensor design with PPP.
In this paper, we discuss the directional dependency problem on the image instrumentation accuracy, specified on the area measurement, and the image resolution. We discuss the evaluation of the relation of the area measurement accuracy in terms of the directional dependency and the image resolution.

## 2. Image Representation using Pseudorandom Pixel Placement

## 2. 1 Pseudorandom Pixel Placement

The objects in an image is represented by pixels, and the position of the photo detector in the image sensor functions as the sampling points of the target object. Here, we call the sampling points in the pixel as "active area" in this paper. The conventional lattice pixel placement (LPP) has the regularly arranged active areas at the lattice positions in the focal plain.


Fig. 2 Pseudorandom pixel placement.


Fig. 3 Definition of pixel parameters.

The pseudorandom pixel placement (PPP) the authors are proposing employs four types of pixels with differently displaced active areas in the pixel, as shown Fig.2(a). Note that all types of the pixels have the completely identical physical bounding dimensions, electric connection terminals and other characteristics, except the positions of the active areas in the pixel. The arrangement of the randomly chosen pixels from four types pixels forms the pseudorandomly arranged active areas as shown in Fig.2(b), and its spatial characteristics can be approximated as the almost completely randomly arranged ${ }^{1)}$. Note that the conventional LPP can be generated by placing one of the four types of pixels in the plain, in other words, PPP is an extension of LPP. It is also notable that the electric connections of the pixels in PPP are identical to those in LPP, and we can read out and process their signals with the conventional technologies, such as raster scan.

### 2.2 Parameter Definitions

In the PPP, we have the following two parameters to define the characteristics of the pixel.

- Aperture ratio, $a[\%]$ : the ratio of the active area size over the pixel size.
- Displacement ratio, $d[\%]$ : the displacement ratio of the active area in the pixel from the center of the pixel. Note that for the practical simulation in this paper, the pixel with the parameters defined above is represented by the pairs of 'actual' pixels. For example, assuming that one pixel for simulation is expressed as $100 \times 100$ 'actual' pixels. In this paper, we call a pixel for simulation composed of several 'actual' pixels as 'virtual pixel'.


Fig. 4 Definition of pixel sampling.

The virtual pixel with the parameter of $a$ and $d$ can be expressed as the pair of the actual pixels (active area pixels and non-active area pixels) as shown Fig.3. Here, $L$ is the number of the actual pixels in one virtual pixel's side, and $A$ is the size (defined as the number of the actual pixels) of the active area in one virtual pixel, and $D$ is the displacement of the active area from the center of the virtual pixel. We can define $a$ and $d$ as follows.

$$
\begin{aligned}
& a=\frac{A^{2}}{L^{2}} \times 100[\%] \\
& d=\frac{D}{L / 2} \times 100[\%]
\end{aligned}
$$

In this paper, we use the unit of [pix] for the number of the actual pixels, and [vpix] for the number of the virtual pixels.

### 2.3 Image Capture Model

Here we define the image capture model using the virtual pixels. The image capture is performed by receiving the photo signal at the active area in each (virtual) pixel, and the digitise of the captured image into binary image is performed based on the signal in each active area with the defined threshold. We assume the threshold of the digitise as 50[\%], and the digitised value of one virtual pixel becomes ' 1 ' if the number of the active area pixels contained in the target object is greater than the half of the number of active area pixels, as shown in Fig. 4.

### 2.4 Image Resolution and Pixel Parameters

The image capture and digitise result will depend on the (virtual) pixel parameters, and it will result in the variations of errors in the area measurement.

Fig. 5 shows two examples of the target objects, the slant rectangular and the amoeba-shape, and their represented images with various (virtual) pixel parameters. Here the number of actual pixels in one virtual pixel side, $L$, is 100 for all the cases, the number of virtual pixels, $N$ is set as $250[\mathrm{vpix}]$ and $500[\mathrm{vpix}]$, and the active area displacement ratio, $d$ of $0[\%]$ and $30[\%]$, with the fixed aperture ratio, $a$ of $25[\%]$. Note that the cases of $d=0[\%]$ correspond to the lattice pixel placement


Fig. 5 Examples of images for various pixel parameters and resolutions. (a)original images, and their magnified images with various parameters, (b) $N=250 / d=0 \%$, (c) $N=250 / d=30 \%$, (d) $N=500 / d=0 \%$, (e) $N=500 / d=30 \%$.

## (LPP).

Generally, the images with the increased resolution $N$ can represent the target object clearly, and the image clarity also depends on the pixel parameters, $a$ and $d$.

## 3. Simulations on Accuracy Evaluation

In this section, we describe the simulation results to evaluate the optimum pixel parameters in terms of the clarity in the area measurement. We also discuss the relation of the resolution and the pixel parameters.

### 3.1 Simulation Conditions

In this section, we discuss on the accuracy in the area measurement using the captured image. As described in subsection 2.1, the image capture situation depends on the direction of the target object in the image, $\theta_{T}$, and it results in that the error in the area measurement depends on $\theta_{T}$.

Here we define the directional dependency on the accuracy in the area measurement as how the accuracy in area measurement does NOT depend on $\theta_{T}$. In case that the error in the area measurement widely changes for various $\theta_{T}$, the accuracy in the area measurement severely depends on $\theta_{T}$. On the other hand, in case that the error is almost constant regardless of $\theta_{T}$, we can measure the area with the same accuracy for various directions of the target object. The variation of the error against $\theta_{T}$ may become large in the case of low resolution, as well as the inappropriate pixel parameters, $a$ and $d$.
To evaluate how the accuracy in the area measurement depends on $\theta_{T}$, we performed the following procedures.
(1) Digitise the target object based on the specified resolution and pixel parameters based on the image capture model described in section 2.3.
(2) Count the number of (virtual) pixels contained in the target object, $N_{p}$.
(3) Calculate $N_{p}\left(\theta_{T}\right)$ for $\theta_{T}=0[\mathrm{deg}], N_{p}(0)$ as a reference.
(4) Calculate $r\left(\theta_{T}\right)=N_{p}\left(\theta_{T}\right) / N_{p}(0)$ for $\theta_{T}=0 \sim$ 180 [deg] with step of $1[\mathrm{deg}]$.
(5) Calculate three statistical measures, the standard deviation (SD), the kurtosis ( $K$ ), and the range $(R)$ for $r\left(\theta_{T}\right)$ in all $\theta_{T}$ values.
The range, $R$ is defined as the difference of the maximum and the minimum values of $r\left(\theta_{T}\right)$.

The kurtosis, $K$, is one of the general statistics mea-

| Table 1 | Simulation parameters of $a$ and $d$. |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $a[\%]$ | $d[\%]$ |  |  |  |  |  |
| 25 | 0 | 10 | 20 | 30 | 40 | 50 |
| 36 | 0 | 10 | 20 | 30 | 40 |  |
| 49 | 0 | 10 | 20 | 30 |  |  |

sures defined as follows.

$$
\begin{gathered}
K=\frac{n(n+1)}{(n-1)(n-2)(n-3)} \sum_{i=1}^{n}\left(\frac{r_{i}-\bar{r}}{\mathrm{SD}}\right)^{4} \\
-\frac{3(n-1)^{2}}{(n-2)(n-3)}
\end{gathered}
$$

Here, $r_{i}, \bar{r}, \mathrm{SD}$ and $n$ are each sample data, the mean of the sample data, the standard deviation, and the number of samples, respectively. $K$ reflects the peak and spreading of frequency distributions of the sample data. The value of $K$ tends to become large in case that the many outliers value samples exist in the sample data for data sets with similar SD, in other words, $K$ is more sensitive to outliers values than SD.

In case of no directional dependency, $N_{p}\left(\theta_{T}\right)$ becomes constant value regardless of $\theta_{T}$, and thus $r\left(\theta_{T}\right)$ becomes 1 for all the cases of $\theta_{T}$. In case with the smaller deviation of the error in the area measurement against $\theta_{T}$, or the smaller directional dependency, will result in the smaller value for these three statistical measures, SD, $K$, and $R$.

Note that the pixel values can be measured as gray scale values in actual image capture process, and the area measurement can be performed with the gray scale pixel values. We discuss these problems in subsection 3.2.

### 3.2 Simulation Results and Discussions

We carried out the simulations to calculate SD, $K$, and $R$ for various pixel parameters by capturing two types of the target objects, the rectangular and the amoeba-shape shown in Fig.5(a). The directional dependency becomes more severe for the objects with line edges, while less severe for the round objects. The two types of objects shown in Fig.5(a) are chosen to represent typical shape of the objects with high and low directional dependencies.

The original images are prepared in $10000 \times 10000$ [pix]. The resolution, or the number of the virtual pixels in one edge, $N$ are set as $N=500,250,125,100[\mathrm{vpix}]$. The aperture ratio, $a$, and the active area displacement ratio, $d$, are set as the values shown in Table 1. Note that upper bound of $d$ is restricted by the size of the active area, or $a$ in order to place the active area inside the virtual pixel.

Table 2 shows the calculated $\mathrm{SD}, K$, and $R$ for var-
ious resolutions, $N$, the aperture ratio, $a$, and the displacement ratio, $d$. The cases those give the small value of $K$ for each resolution in the rectangular object case is indicated with the underline.

From Table 2(a), we can find the pair of the parameters of $(a, d)=(25 \%, 40 \%)$, which is indicated with bold face and underlines, that give the small $K$ for all the resolutions, and this pair of the parameters also gives the small SD and $R$, although it does not give the minimum values.

We can also find that $(N, a, d)=(250,25 \%, 40 \%)$ gives the similar value of SD and $R$ with $(N, a, d)=$ $(500,25 \%, 0 \%)$, as well as $(N, a, d)=(125,25 \%, 40 \%)$ with $(N, a, d)=(250,25 \%, 0 \%)$.

This result indicates that the optimum pixel parameter of $(a, d)=(25 \%, 40 \%)$ has the comparable directional dependency on the accuracy with $d=0 \%$, or lattice pixel placement (LPP) with twice $(\times 2)$ higher resolution with the same active pixel area size. The pair of $(N, a, d)=(100,25,40)$ gives smaller values of SD and $R$ than $(N, a, d)=(125,25,0)$, or LPP with less than twice larger resolution.

Note that the pixel parameter of $(a, d)=(36 \%, 40 \%)$ also gives the similar values of $\mathrm{SD}, K$, and $R$ to those with $(a, d)=(25 \%, 40 \%)$. The optimum values of $(a, d)$ may exist among $(a, d)=(25 \%, 40 \%)$ and $(36 \%, 40 \%)$, and detailed optimum values should be evaluated and discussed in our future works.

These results show that PPP with the optimum pixel parameter can measure the area of the rectangular with half $(1 / 2)$ resolution of the LPP.

In Table 2(b), SD, $K$, and $R$ have small deviations against the pixel parameters in each resolution. This is because the amoeba-shape have no line edge, and basically have the small directional dependency. With the same terms of evaluations for the rectangular, the pair of $(a, d)=(25 \%, 40 \%)$, which is indicated with bold face and underlines, gives the smaller SD and $R$ than the pair of $(a, d)=(25 \%, 0 \%)$ or LPP with twice higher resolutions.

As discussed above, the PPP with the optimum parameter of $(a, d)=(25 \%, 40 \%)$ gives the comparable accuracy with half resolution of LPP for various types of objects.

Note that the optimum values of $(a, d)$ should be evaluated and discussed for various types of target object shapes in our future works.

As indicated in subsection 3.1, the pixel values can be measured as gray scale values in actual image cap-
ture process, and the area measurement can be performed with the gray scale pixel values. We can treat the directional dependency caused by pixel placement independently of pixel digitize ${ }^{122)}$; the directional dependency can be decreased by optimum pseudorandom pixel placement for both binary image and gray scale image. The detailed area measurement with taking the gray scale pixel values into consideration will be evaluated and discussed in our future works.

### 3.3 Conclusion

In this paper, we proposed the pseudorandom pixel placement (PPP) for reducing the directional dependency on the accuracy in the area measurement. We carried out the simulations to evaluate the directional dependency decrease effect for various pixel parameters, and the simulation results show that the PPP with the optimum parameter of $(a, d)=(25 \%, 40 \%)$ gives the comparable accuracy with half $(1 / 2)$ resolution of LPP for various types of objects.

The detailed optimum parameters of $(a, d)$, as well as area measurement with using gray scale values, will be evaluated and discussed in our future works.

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Table 2 Calculated SD, $K$, and $R$ for various pixel parameters. (a)rectangular and (b)amoeba-shape.
(a)

| $\begin{gathered} N= \\ 500 \end{gathered}$ | $a$ | 25\% |  |  |  |  |  | $36 \%$ |  |  |  |  | 49\% |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $d$ | 0\% | 10\% | 20\% | 30\% | 40\% | 50\% | 0\% | 10\% | 20\% | 30\% | 40\% | 0\% | 10\% | 20\% | 30\% |
|  | $\mathrm{SD}\left[\times 10^{-3}\right]$ | 0.48 | 0.41 | 0.22 | 0.27 | 0.29 | 0.32 | 0.48 | 0.41 | 0.27 | 0.28 | 0.28 | 0.48 | 0.42 | 0.26 | 0.29 |
|  | K | 144.2 | 109.5 | 32.2 | 29.1 | $\underline{28.3}$ | $\underline{12.9}$ | 146.2 | 95.1 | 49.0 | 33.5 | $\underline{26.7}$ | 147.1 | 109.1 | 58.4 | 28.6 |
|  | $R\left[\times 10^{-3}\right]$ | 6.40 | 5.36 | 2.40 | 2.82 | 3.11 | 3.13 | 6.40 | 5.16 | 3.09 | 3.07 | 2.93 | 6.40 | 5.29 | 3.16 | 3.04 |
| $\begin{gathered} N= \\ 250 \end{gathered}$ | $a$ | 25\% |  |  |  |  |  | $36 \%$ |  |  |  |  | 49\% |  |  |  |
|  | d | 0\% | 10\% | 20\% | 30\% | 40\% | 50\% | 0\% | 10\% | 20\% | 30\% | 40\% | 0\% | 10\% | 20\% | 30\% |
|  | $\mathrm{SD}\left[\times 10^{-3}\right]$ | 1.72 | 1.07 | 0.59 | 0.58 | 0.56 | 0.66 | 1.72 | 1.07 | 0.59 | 0.58 | 0.56 | 1.72 | 1.05 | 0.57 | 0.60 |
|  | K | 51.7 | 77.3 | 63.9 | 35.2 | $\underline{23.0}$ | 40.5 | 51.8 | 77.7 | 64.7 | 33.9 | $\underline{22.0}$ | 51.8 | 72.2 | 82.6 | 53.9 |
|  | $R\left[\times 10^{-3}\right]$ | 14.04 | 12.00 | 7.20 | 6.76 | 5.96 | 7.47 | 14.04 | 12.00 | 7.20 | 6.67 | 5.87 | 14.04 | 11.73 | 7.11 | 7.82 |
| $\begin{gathered} N= \\ 125 \end{gathered}$ | $a$ | 25\% |  |  |  |  |  | $36 \%$ |  |  |  |  | 49\% |  |  |  |
|  | d | 0\% | 10\% | 20\% | 30\% | 40\% | 50\% | 0\% | 10\% | 20\% | 30\% | 40\% | 0\% | 10\% | 20\% | 30\% |
|  | $\mathrm{SD}\left[\times 10^{-3}\right]$ | 2.17 | 2.11 | 2.06 | 2.06 | 2.10 | 2.11 | 2.17 | 2.11 | 2.06 | 2.06 | 2.09 | 2.17 | 2.09 | 2.06 | 2.09 |
|  | K | $\underline{24.4}$ | 25.8 | 26.5 | 27.2 | $\underline{24.6}$ | $\underline{23.8}$ | $\underline{24.3}$ | 25.8 | 26.4 | 27.1 | $\underline{24.7}$ | $\underline{24.2}$ | 26.7 | 26.8 | $\underline{24.6}$ |
|  | $R\left[\times 10^{-3}\right]$ | 21.69 | 21.69 | 17.78 | 17.07 | 17.42 | 17.78 | 21.69 | 21.69 | 17.78 | 17.07 | 17.42 | 21.69 | 21.69 | 18.49 | 18.13 |
| $\begin{gathered} N= \\ 100 \end{gathered}$ | $a$ | 25\% |  |  |  |  |  | 36\% |  |  |  |  | 49\% |  |  |  |
|  | d | 0\% | 10\% | 20\% | 30\% | 40\% | 50\% | 0\% | 10\% | 20\% | 30\% | 40\% | 0\% | 10\% | 20\% | 30\% |
|  | $\mathrm{SD}\left[\times 10^{-3}\right]$ | 2.18 | 2.16 | 2.17 | 2.21 | 2.24 | 2.30 | 2.19 | 2.16 | 2.17 | 2.21 | 2.24 | 2.19 | 2.14 | 2.20 | 2.17 |
|  | K | 31.9 | 33.3 | 32.5 | 28.9 | 26.0 | $\underline{25.3}$ | 31.7 | 33.8 | 32.6 | 29.0 | $\underline{25.9}$ | 31.7 | 34.4 | 30.7 | 31.1 |
|  | $R\left[\times 10^{-3}\right]$ | 18.89 | 18.89 | 19.44 | 19.44 | 20.00 | 19.44 | 18.89 | 18.89 | 19.44 | 19.44 | 20.00 | 18.89 | 18.89 | 19.44 | 20.56 |

(b)

| $\begin{gathered} N= \\ 500 \end{gathered}$ | $a$ | 25\% |  |  |  |  |  | 36\% |  |  |  |  | 49\% |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | d | 0\% | 10\% | 20\% | 30\% | 40\% | 50\% | 0\% | 10\% | 20\% | 30\% | 40\% | 0\% | 10\% | 20\% | 30\% |
|  | $\mathrm{SD}\left[\times 10^{-3}\right]$ | 1.76 | 1.75 | 1.77 | 1.77 | 1.75 | 1.78 | 1.76 | 1.75 | 1.77 | 1.77 | 1.75 | 1.76 | 1.76 | 1.76 | 1.77 |
|  | K | 5.7 | 5.8 | 5.7 | 5.6 | 5.9 | 5.7 | 5.7 | 5.8 | 5.7 | 5.6 | 5.9 | 5.7 | 5.7 | 5.5 | 5.8 |
|  | $R\left[\times 10^{-3}\right]$ | 7.59 | 7.48 | 7.51 | 7.50 | 7.42 | 7.69 | 7.58 | 7.47 | 7.51 | 7.48 | 7.47 | 7.57 | 7.54 | 7.41 | 7.73 |
| $\begin{aligned} & N= \\ & 250 \end{aligned}$ | $a$ | 25\% |  |  |  |  |  | 36\% |  |  |  |  | 49\% |  |  |  |
|  | $d$ | 0\% | 10\% | 20\% | 30\% | 40\% | 50\% | 0\% | 10\% | 20\% | 30\% | 40\% | 0\% | 10\% | 20\% | 30\% |
|  | $\mathrm{SD}\left[\times 10^{-3}\right]$ | 1.82 | 1.83 | 1.81 | 1.82 | 1.82 | 1.83 | 1.82 | 1.83 | 1.82 | 1.83 | 1.80 | 1.82 | 1.80 | 1.85 | 1.81 |
|  | K | 4.8 | 4.9 | 4.9 | 4.8 | 4.8 | 5.00 | 4.9 | 5.0 | 4.9 | 5.1 | 4.2 | 4.9 | 5.0 | 4.7 | 4.7 |
|  | $R\left[\times 10^{-3}\right]$ | 8.40 | 8.68 | 8.12 | 8.01 | 8.46 | 8.62 | 8.40 | 8.85 | 8.06 | 8.51 | 8.06 | 8.40 | 8.34 | 8.57 | 8.68 |
| $\begin{gathered} N= \\ 125 \end{gathered}$ | $a$ | 25\% |  |  |  |  |  | 36\% |  |  |  |  | 49\% |  |  |  |
|  | $d$ | 0\% | 10\% | 20\% | 30\% | 40\% | 50\% | 0\% | 10\% | 20\% | 30\% | 40\% | 0\% | 10\% | 20\% | 30\% |
|  | $\mathrm{SD}\left[\times 10^{-3}\right]$ | 2.05 | 2.08 | 1.98 | 2.09 | 1.98 | 2.09 | 2.04 | 2.09 | 1.99 | 2.08 | 1.97 | 2.03 | 2.01 | 2.10 | 2.07 |
|  | K | 3.5 | 4.2 | 3.9 | 2.9 | 4.1 | 3.2 | 3.5 | 4.2 | 4.0 | 2.8 | 4.0 | 3.5 | 3.8 | 3.7 | 3.9 |
|  | $R\left[\times 10^{-3}\right]$ | 9.92 | 11.06 | 10.38 | 10.38 | 11.74 | 11.51 | 9.92 | 11.06 | 10.38 | 10.15 | 11.51 | 9.92 | 10.38 | 10.84 | 11.74 |
| $\begin{gathered} N= \\ 100 \end{gathered}$ | $a$ | 25\% |  |  |  |  |  | 36\% |  |  |  |  | 49\% |  |  |  |
|  | d | 0\% | 10\% | 20\% | 30\% | 40\% | 50\% | 0\% | 10\% | 20\% | 30\% | 40\% | 0\% | 10\% | 20\% | 30\% |
|  | $\mathrm{SD}\left[\times 10^{-3}\right]$ | 2.29 | 2.21 | 2.28 | 2.21 | 2.18 | 2.10 | 2.28 | 2.21 | 2.28 | 2.22 | 2.19 | 2.29 | 2.27 | 2.20 | 2.28 |
|  | K | 0.9 | 1.3 | 1.8 | 1.9 | 2.4 | 2.9 | 0.95 | 1.3 | 1.8 | 1.8 | 2.3 | 0.9 | 1.1 | 1.7 | 2.0 |
|  | $R\left[\times 10^{-3}\right]$ | 11.64 | 11.98 | 13.04 | 13.04 | 13.06 | 12.68 | 11.64 | 11.98 | 13.04 | 13.04 | 13.06 | 11.64 | 11.99 | 13.05 | 13.07 |


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