Effect of Providing Traffic Information Estimated by a Stochastic Network Equilibrium Model with Stochastic Demand

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Abstract

To estimate travel times through road networks, in this study, we assume a stochastic demand and formulate a stochastic network equilibrium model whose travel times, flows, and demands are stochastic. This model enables us to examine network reliability under stochastic circumstances and to evaluate the effect of providing traffic information on travel times. For traffic information, we focus on travel time information and propose methods to evaluate the effect of providing that information. To examine the feasibility and validity of the proposed model and methods, we apply them to a simple network and the real road network of Kanazawa, Japan. The results indicate that providing ambulance drivers in Kanazawa with travel time information leads to an average reduction in travel time of approximately three minutes.

Keywords:
stochastic network equilibrium, emergency vehicles, travel time information provision

1 Introduction

Numerous studies have been undertaken to model the benefits of traffic information. To assess the effect of providing traffic information through systems such as intelligent transportation systems (ITS) or advanced traveler information systems (ATIS), it is vital to evaluate the uncertainty of traffic networks and to analyze their reliability. This is because traffic information is only meaningful when the traffic conditions are uncertain or risky. If drivers have a complete knowledge of traffic conditions and can predict exact travel times, there is no benefit from ITS, ATIS, or traffic information. Thus, to determine the effect of traffic information, traffic uncertainty should be quantitatively analyzed. A network equilibrium model is a theoretical approach to determining traffic uncertainty that allows the probability distributions of travel times or traffic flows to be estimated.

To date, many studies have been conducted to model the effect of a traffic information provision in the network equilibrium framework (e.g. Arnott et al., 1991; Yang, 1998; Jha et al., 1998; Yang & Meng, 2001; Lo & Szeto, 2002; Yin & Yang, 2003; Yang & Huang, 2004; de Palma & Picard, 2006; Huang & Li, 2007; Huang et al., 2008 & 2011; Wu et al., 2010; de Palma et al., 2012). User equilibrium (UE), stochastic user equilibrium (SUE), and system optimum (SO) are used in most of these studies. The total travel cost is minimized at the state of SO, but SO is not necessarily achieved, even if the exact traffic information is provided. This is because, in real life, drivers typically choose the routes voluntarily. UE is a benchmark when traffic information is provided to drivers with voluntary route choice.

SUE is one of the most important forms of network equilibria. With SUE, the (route) utility in the route choice contains an error term, but the interpretation of the error term is debated. In most SUE cases, the error term of random utility route choice is given exogenously, and it does not seem to reflect the probability distribution of travel time, which is determined endogenously through network equilibrium. Therefore, the error term in SUE should be interpreted as a “perceptual” error, rather than a travel time distribution (e.g. Hazelton, 1998).

When users with little travel experience are dominant in the network, SUE (with given error terms) could be applicable to model the uncertainty of the traffic network. This is because the perceptions of naïve users do not necessarily reflect actual traffic conditions and may form unilaterally. In this study, we discuss the case with users who are familiar with traffic conditions, e.g. commuters. In such a case, the conventional SUE is not necessarily appropriate to model the effect of a traffic information provision, since the recognition of experienced users is close to the actual travel time distribution. It is natural to assume that the drivers know the travel time distribution in this case. Thus, a stochastic network equilibrium with stochastic flows is desirable to model the stochastic travel time variability with experienced drivers.

Many groups have studied the network equilibrium with stochastic flows or travel times. For example, Mirchandani & Soroush (1987) assumed that free-flow travel time is random and proposed a network equilibrium model with probabilistic travel times. Arnott et al. (1991) and Chen et al. (2002) introduced random capacity into the network equilibrium. Lo and colleagues (Lo & Tung, 2003; Lo et al., 2006) formulated a probabilistic user equilibrium model based on capacity reliability. More recently, Nie (2011) proposed a percentile user equilibrium with random capacity. The network equilibrium models of Yin and colleagues (Yin & Ieda, 2001; Yin et al., 2004) and Watling (2006) assumed stochastic, normally distributed travel times, whereas Chen & Zhou (2010) assumed log-normally distributed travel times. Watling (2002) considered stochastic route choice, and developed a second-order stochastic network equilibrium with stochastic flows.

A main cause of network uncertainty is variation in travel demand; thus, stochastic demand should be incorporated into network equilibrium models. Shao et al. (2006) and Siu & Lo (2008) introduced stochastic demands into network equilibrium models and explicitly considered stochastic flows. Lam et al. (2008) and Sumalee et al. (2010) considered network flow under stochastic supply and demand, and examined the impacts of adverse weather conditions. Zhang et al. (2011), who similarly considered stochastic supply and demand, introduced the expected residual minimization into stochastic-flow network equilibrium. Nakayama & Watling (2014) proposed stochastic network equilibrium models with stochastic flows, in which stochastic travel time, network flow, and demand are treated consistently. However, they assumed that the stochastic travel demands are independent among origin-destination (OD) pairs.

In this study, we improve upon the model of Nakayama & Watling (2014), alleviating the independent demand and other assumptions, and formulate a stochastic network equilibrium model under stochastic demands. Using the proposed stochastic network equilibrium model, we develop methods to evaluate the effect of providing travel time information. Then, we apply the model and methods to a simple network and the real road network of Kanazawa, Japan, and examine their applicability and validity.

2. Assumptions for Drivers and Notation

The notation used in this study is as follows:

- I = the number of OD pairs
- A = the total number of links
- J_i = the number of routes between OD pair i
- J = the total number of all routes (= $J_1 + J_2 + \cdots + J_I$)
- K = the total number of latent drivers
- $\delta_{a,ij}$ = link route incidence variable ($\delta_{a,ij}$ = 1 if link a is part of route j between OD pair i; 0 otherwise)
- $\mathbf{\Delta}$ = the link route incidence matrix whose components are $\delta_{a,ij}$
- $\mathbf{\Gamma}$ = the OD route incidence matrix whose components are binary variables of indicating which OD pair connects the route
- N_{ik} = the random variable that determines whether driver k travels between OD pair i (N_{ik} = 1 if driver k takes OD pair i; 0 otherwise)
- Q_i = the random variable of travel demand between OD pair i
- $\mathbf{Q}$ = the random vector of travel demands = $(Q_0, Q_1, Q_2, \ldots, Q_I)^{\mathrm{T}}$, ($Q_0$ = the number of drivers who make no trip)
- $\mathbf{q}_l$ = the actual demand vector on day l
- μ_i = the mean of demand between OD pair i
- $\boldsymbol{\mu}$ = the mean vector of travel demands

σ_i^2 = the variance of demand between OD pair i
$\sigma_{ii'}$ = the covariance of Q_i and $Q_{i'}$
$\mathbf{\Sigma}$ = the variance-covariance matrix of travel demands
X_a = the random variable of flow on link a
$\mathbf{X}$ = the random vector of all link flows = $(X_1, X_2,\ldots, X_A)^{\mathrm{T}}$
$f_{Xa}(\cdot)$ = the probability density function of flow on link a
Y_{ij} = the random variable of flow on route j between OD pair i
$\mathbf{Y}$ = the random vector of all route flows = $(Y_{11}, Y_{12},\ldots, Y_{IJ_I})^{\mathrm{T}}$
m_a = the mean flow on link a
s_a^2 = the variance of flow on link a
$s_{a,a'}$ = the covariance of flows on links a and a'
m_{ij} = the mean flow on route j between OD pair i
s_{ij}^2 = the variance of flow on route j between OD pair i
$s_{ij,i'j'}$ = the covariance of flows on route j between OD pair i and route j' between OD pair i'
$\mathbf{m}$ = the mean vector of route flows
$\mathbf{S}$ = the variance-covariance matrix of route flows
$t_a(\cdot)$ = the (deterministic or standard) travel time function on link a
$\mathbf{t}(\cdot)$ = the vector function for travel times = $(t_1(\cdot), t_2(\cdot),\ldots, t_A(\cdot))^{\mathrm{T}}$
t_{ij} = the travel time on route j between OD pair i
$\bar{t}_{ij}$ = the mean travel time on route j between OD pair i
T_a = the random variable of travel time on link a
T_{ij} = the random variable of travel time on route j between OD pair i
t_{ijl} = the (realized) travel time on route j between OD pair i for day l
τ_a = the free-flow travel time on link a
γ_a = the capacity on link a
α, β = travel time function parameters
c_{ij} = efficient travel time on route j between OD pair i
$\mathbf{c}(\cdot)$ = the vector function for efficient travel times = $(c_{11}(\cdot), c_{12}(\cdot),\ldots, c_{IJ_I}(\cdot))^{\mathrm{T}}$
ω_{ij} = standard deviation (SD) of travel time on route j between OD pair i
$\boldsymbol{\varsigma}$ = the vector of variables on the minimum mean route travel times
r_{ij} = the route choice proportion of uninformed drivers on route j between OD pair i.
$\mathbf{r}$ = the vector of route choice proportions of uninformed drivers = $(r_{11}, r_{12},\ldots, r_{IJ_I})^{\mathrm{T}}$
P_{ij} = the random variable of the route choice proportion of informed drivers on route j between OD pair i
$\mathbf{P}$ = the random vector of route choice proportions of informed drivers = $(P_{11}, P_{12},\ldots, P_{IJ_I})^{\mathrm{T}}$
p_{ij} = the realized value of P_{ij}
$\mathbf{p}_l$ = the vector of route choice proportions of informed drivers on day l
π = the proportion of informed drivers for all drivers
η = the common risk attitude parameter
E[X] = the mean of random variable of X

Var[X] = the variance of random variable of X

Cov[X,Y] = the covariance of the two random variables of X and Y

Pr[·] = the probability calculation operator

$\mathbb{N}[\mathbf{m}, \mathbf{S}]$ = the normal distribution whose mean and variance are **m** and **S**

$\langle \mathbf{x}, \mathbf{y} \rangle$ = the inner product of **x** and **y**

diag(**x**) = the diagonal matrix whose components are **x**

min[·] = the operator that returns the minimum

$\mathbf{ln}(\mathbf{x}) = (\ln x_{11}, \ln x_{12}, \ldots, \ln x_{IJ_I})^{\mathrm{T}}$

0 = the null matrix or vector

I = the unit vector

$^{\mathrm{T}}$ = the transition for vectors or matrices.

$\mathbb{i}$ = the imaginary unit

R_+^J = non-negative orthant of the J-dimensional space

For this model, we make the following assumptions:

A1. Drivers, including latent drivers (see Section 3.1), are rational, homogenous, and mutually independent.

A2. Each latent driver randomly determines whether to make a trip or randomly changes his destination due to exogenous factors.

A3. Route choice is made under the stationary condition of stochastic network equilibrium.

A4. The route choice proportion is given *deterministically* by the random utility discrete choice model.

In this study, the travel demands are stochastic; therefore, flows and travel times are random. However, due to Assumption A4, route choice remains deterministic in this study. The random utility route choice model gives the "proportion" of choosing a given route rather than the probability. The error term in the random utility model is regarded as an unobserved factor and is deterministically distributed. As discussed below, the route choice proportions are determined through the network equilibrium mechanism. Although route choice is deterministic, the decision to make a trip is random, according to Assumption A2. Therefore, traffic flows are random.

Here, we focus on travel time information as traffic information. Two types of drivers are assumed: informed and uninformed drivers. Uninformed drivers have no traffic information and, therefore, cannot predict a priori the travel time for the route they have chosen. However, the drivers are assumed to be familiar with the daily traffic conditions, and this leads to a fifth assumption.

A5. Uninformed drivers know "past" travel times from which they form probability distributions of route travel times.

In this study, the stationary state is assumed, and route choice is made under the stationary state, as is mentioned in Assumption A3. The impact of the most recent travel time experiences may be greater in a non-stationary state. However, each travel time experience should be treated equally, regardless of whether it is new or old, in the stationary state. This is because each travel time is realized from the same (or stationary) travel time distribution.

Consistent with Assumption A5, drivers take into consideration the probability distribution of route travel times for route choice. All drivers are risk averse and, when deciding which route to take, they make allowance for a safety margin to avoid arriving late at their destination. There are several methods to quantify uncertainty (or variability) in travel time. Here, we adopt the efficient travel time proposed by Hall (1983). The efficient travel time comprises a mean travel time and safety margin. For simplicity, we denote the efficient travel time c_{ij} as $\bar{t}_{ij} + \eta\,\omega_{ij}$. Throughout this study, the link travel time is random, and is a function of random link flow. Therefore, $\bar{t}_{ij} = \mathrm{E}[\sum_{a=1}^{A} \delta_{a,ij}\, t_a(X_a)]$ and the efficient travel time is flow-dependent. An effective method to calculate the efficient travel time is discussed in Section 4. We also used a sixth assumption as follows:

A6. Each uninformed driver chooses a route deterministically based on efficient travel times.

Conversely, informed drivers possess traffic information provided by the road manager. Unless otherwise noted, traffic information is assumed to be significantly accurate. If the travel time information is inaccurate, we apply the same method to choose a route, as described above for uninformed drivers. Uninformed drivers determine a route travel time distribution solely based on their past travel times, whereas informed drivers with inaccurate travel time information do so based on inaccurate travel time information and past travel times. Furthermore, informed drivers with inaccurate travel time information have to incorporate a safety margin because they do not know a priori the exact travel time for their trip, and their travel cost should be the efficient travel time, just as for uninformed drivers. Therefore, informed drivers with inaccurate travel times can be modeled in the same manner as uninformed drivers. In this study, we focus on the case in which the travel time information is accurate. However, the quality of information is another research subject that should be investigated, e.g. Ben-Elia (2013). Since informed drivers with accurate a priori travel time information do not need a safety margin, we are led to the following seventh assumption:

A7. Informed drivers choose routes solely based on travel times provided by the road network manager.

In other words, the variable for the informed drivers' route choice model is the route travel time provided by the road manager.

3. Demand and Route Flow Distributions

3.1 Stochastic demand distributions

We generalize the concept by allowing a *random* change of the OD pair, and introduce a hypothetical OD pair. The notation $N_{0k} = 1$, means that driver k makes no trip. It is assumed for simplicity that $\mathbf{N}_k = (N_{0k}, N_{1k},\ldots, N_{Ik})^{\mathrm{T}}$ (k = 1, 2,…, K) is exogenously given in this study, because $\mathbf{Q} = \sum_{k=1}^{K} \mathbf{N}_k$, the demand vector, $\mathbf{Q}$, is also exogenous. According to Assumption A1, $\mathbf{N}_k$ is independent and identically distributed among drivers. Note that $N_{0k}, N_{1k},\ldots, N_{Ik}$ are not necessarily independent within the same driver, and the demands $(Q_0, Q_1,\ldots, Q_I)$ may be correlated.

Consider the behavior of a single (latent) driver. A simple behavior is that the driver changes the OD pair (or destination) at random, with each OD pair having a fixed probability of being chosen. The sum of these behaviors leads to multinomially distributed OD demands. This is just a simple example; in reality however, it is more complicated, so we consider a more general case. Instead of assuming a specified behavior, we define the mean vector and variance-covariance matrix of N_{ik} as follows:

$$\boldsymbol{\mu}_k = \begin{pmatrix} \mathrm{E}[N_{0k}] \\ \mathrm{E}[N_{1k}] \\ \vdots \\ \mathrm{E}[N_{Ik}] \end{pmatrix}, \tag{1}$$

$$\boldsymbol{\Sigma}_k = \begin{pmatrix} \mathrm{Var}[N_{0k}] & \mathrm{Cov}[N_{0k}, N_{1k}] & \cdots & \mathrm{Cov}[N_{0k}, N_{Ik}] \\ \mathrm{Cov}[N_{1k}, N_{0k}] & \mathrm{Var}[N_{1k}] & \cdots & \mathrm{Cov}[N_{1k}, N_{Ik}] \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{Cov}[N_{Ik}, N_{0k}] & \mathrm{Cov}[N_{Ik}, N_{1k}] & \cdots & \mathrm{Var}[N_{Ik}] \end{pmatrix}. \tag{2}$$

Clearly, Eqs. (1) and (2) include those of multinomially distributed and more complicated cases.

As per Assumption A1, each latent driver is independent, so $\boldsymbol{\mu} = \sum_{k=1}^{K} \boldsymbol{\mu}_k$ and $\boldsymbol{\Sigma} = \sum_{k=1}^{K} \boldsymbol{\Sigma}_k$.

The central limit theorem for independent multivariate random variables can be applied to $\mathbf{Q}$. Because N_{ik} is binary, $\Pr[|N_{0k}| \le 1, |N_{1k}| \le 1,\ldots] = 1$, and the random vector $\mathbf{N}_k = (N_{0k}, N_{1k},\ldots)^{\mathrm{T}}$ is uniformly bounded. Nakayama & Watling (2014) examine the central limit theorem applied to multivariate random route flows. The same method can be applied to these random OD demands.

For convenience, we introduce the variable W_k, which represents the weighted sum of N_{ik}. Let $W_k = \boldsymbol{\kappa}^{\mathrm{T}} \mathbf{N}_k$, where $\boldsymbol{\kappa}$ is the weight vector and $\boldsymbol{\kappa} = (\kappa_0, \kappa_1 \ldots \kappa_I)^{\mathrm{T}}$. Using W_k, the vector $\mathbf{N}_k$ can be treated as a scalar, which means that the standard central limit theorem is applicable.

First, we show that the sum of W_k converges to a (univariate) normal distribution. Next, we show that this sum is identical to the asymptotically and normally distributed $\mathbf{Q}$. Because $\mathbf{N}_k$ is uniformly bounded, W_k is also uniformly bounded when $\boldsymbol{\kappa}$ is in the finite sphere. Also, $\mathrm{E}[W_k] = \mathrm{E}[\boldsymbol{\kappa}^{\mathrm{T}} \mathbf{N}_k] = \boldsymbol{\kappa}^{\mathrm{T}} \boldsymbol{\mu}_k$ and $\mathrm{Var}[W_k] = \mathrm{Var}[\boldsymbol{\kappa}^{\mathrm{T}} \mathbf{N}_k] = \boldsymbol{\kappa}^{\mathrm{T}} \boldsymbol{\Sigma}_k \boldsymbol{\kappa}$. Clearly, $\mathrm{Var}[\Sigma_{k=1}^{K} W_k] \to \infty$ as $K \to \infty$ ($\boldsymbol{\kappa} \ne \mathbf{0}$) because $\mathrm{Var}[W_k] > 0$. If the standardized random variables U_0, $U_1,\ldots$, whose mean and variance are 0 and 1, respectively, are independent and uniformly bounded, then $(1/\sqrt{K}) \Sigma_{k=0}^{K} U_k \to \mathbb{N}[0, 1]$ as $K \to \infty$ (Loève, 1977, p. 289). As $K \to \infty$,

$$\frac{\sum_{k=1}^{K} W_k - \sum_{k=1}^{K} \mathrm{E}[W_k]}{\sqrt{\sum_{k=1}^{K} \mathrm{Var}[W_k]}} = \frac{\sum_{k=1}^{K} W_k - \boldsymbol{\kappa}^{\mathrm{T}}\boldsymbol{\mu}}{\sqrt{\boldsymbol{\kappa}^{\mathrm{T}}\boldsymbol{\Sigma}\boldsymbol{\kappa}}} \longrightarrow \mathbb{N}[0,1] \tag{3}$$

because $\boldsymbol{\mu} = \sum_{k=1}^{K} \boldsymbol{\mu}_k$ and $\boldsymbol{\Sigma} = \sum_{k=1}^{K} \boldsymbol{\Sigma}_k$. Therefore, $\sum_{k=1}^{K} W_k \to \mathbb{N}[\boldsymbol{\kappa}_i^{\mathrm{T}}\boldsymbol{\mu}_i, \boldsymbol{\kappa}_i^{\mathrm{T}}\boldsymbol{\Sigma}_i\boldsymbol{\kappa}_i]$ (Stuart & Ord, 1994, p. 512). We now consider the convergence of random vectors of demand. To denote the convergence of random vectors, $\mathbf{Q}_K = \sum_{k=1}^{K} \mathbf{N}_k - \boldsymbol{\mu}$ is used instead of $\mathbf{Q}$. In this case,

$$\boldsymbol{\kappa}^T \mathbf{Q}_\nu = \sum_{k=1}^{K} W_k - \boldsymbol{\kappa}^T \boldsymbol{\mu} \longrightarrow \mathbb{N}[\mathbf{0}, \boldsymbol{\kappa}^T \boldsymbol{\Sigma}\boldsymbol{\kappa}]. \tag{4}$$

According to the Cramér–Wold device (Billingsley, 1995, p. 380), a necessary and sufficient condition for $\mathbf{Q}_K \to \mathbf{V}$ is $\sum_{k=1}^{K} \sum_{i=0}^{I} \kappa_i Q_{ik} \to \sum_{i=0}^{I} \kappa_i V_i$ for each $\boldsymbol{\kappa}$ in a finite $(I+1)$-dimensional sphere, where $\mathbf{V}$ = $(V_1, V_2, \ldots)$. If $\mathbf{V} = \mathbb{N}[\mathbf{0}, \boldsymbol{\Sigma}]$, then $\sum_{i=0}^{I} \kappa_i V_i$ = $\mathbb{N}[\mathbf{0}, \boldsymbol{\kappa}^T \boldsymbol{\Sigma}\boldsymbol{\kappa}]$ (Stuart & Ord, 1994, p. 512). As Eq. (4) indicates, $\sum_{i=0}^{I} \kappa_i V_i$ = $\boldsymbol{\kappa}^T \mathbf{Q}_\nu \to \mathbb{N}[\mathbf{0}, \boldsymbol{\kappa}^T \boldsymbol{\Sigma}\boldsymbol{\kappa}]$. We confirm that $\mathbf{Q}_\nu \to \mathbf{V}$ because $\sum_{k=1}^{K} \sum_{i=0}^{I} \kappa_i Q_{ik} \to \sum_{i=0}^{I} \kappa_i U_i \sim \mathbb{N}[\mathbf{0}, \boldsymbol{\kappa}^T \boldsymbol{\Sigma}\boldsymbol{\kappa}]$. Therefore, as $K \to \infty$,

$$\mathbf{Q} \longrightarrow \mathbb{N}[\boldsymbol{\mu}, \boldsymbol{\Sigma}]. \tag{5}$$

Thus, when demand is sufficiently large, the distribution of travel demand approximately follows the multivariate normal distribution.

3.2 Route flow distribution

As Assumption A4 states, the route choice proportion is given deterministically in the random utility discrete choice model. In addition, the route choice differs for informed and uninformed drivers.

The chosen OD pair (or destination), including no travel, is determined randomly and exogenously, and the travel demand is normally distributed, as shown in Section 3.1. When all drivers are uninformed, the random flow on route j between OD pair i is $\sum_{k=1}^{K} r_{ij} N_{ik} = r_{ij} Q_i$ (i = 1, 2,…, I) due to Assumption A4. Route choice is not made for the drivers who do not travel, and $\sum_{k=1}^{K} N_{0k} = Q_0$. Although some latent drivers make no trips, route/link flows are illustrated from the revealed demands (Q_1, Q_2,…, Q_I) in the remainder of this section, because Q_0 does not contribute to the revealed route/link flows.

The mean of $r_{ij} Q_i$ is $r_{ij} \mu_i$ and the variance is $r_{ij} \sigma_i^2$ (i = 1, 2,…, I), because the driver's behavior is independent. In this case, $Y_{ij} = r_{ij} Q_i \sim \mathbb{N}[r_{ij}\mu_i, r_{ij}\sigma_i^2]$ (i = 1, 2,…, I), since Q_i is asymptotically normally distributed and r_{ij} is deterministic, as stated above. Note that $Q_0 \sim \mathbb{N}[\mu_0, \sigma_0^2]$.

An example of a simple case is as follows. The network has two OD pairs and each OD pair has two routes. Set r_{11} = 0.4, r_{12} = 0.6, r_{21} = 0.3, r_{22} = 0.7, and K =100, as shown in Fig. 1. Thus, $r_{11} + r_{12} = 1$ and $r_{21} + r_{22} = 1$. The route choice of each latent driver is deterministic as Assumption A4 mentions, even if they randomly determine whether to make a trip. The driver does not have an incentive to switch the route randomly and then continue to take that route. Thus, each latent driver determines the fixed route when making a trip. Furthermore, the route choice proportions, r_{11} (= 0.12),

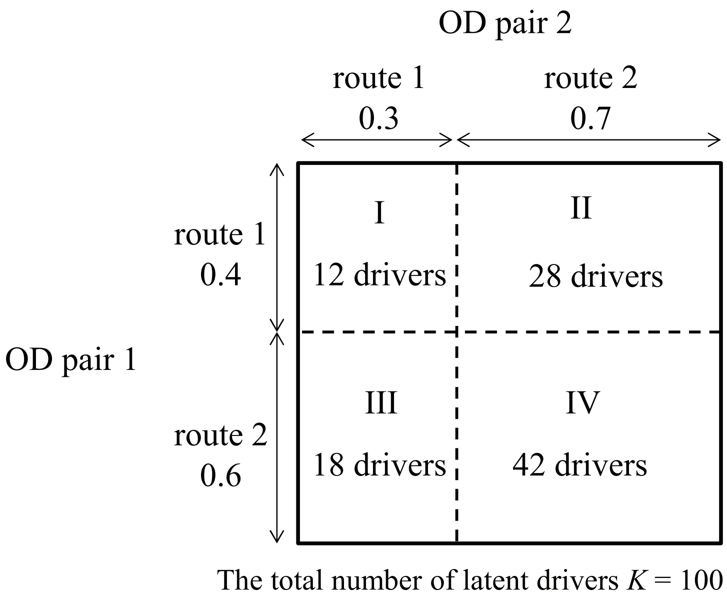

Fig. 1 The latent drivers' assignments in the 2-OD-4-route network

r_{12} (= 0.28), r_{21} (= 0.18) and r_{22} (= 0.42), are given endogenously through the equilibrium mechanism, and are illustrated in Fig. 1. The drivers in region II of Fig. 1 choose route 1 unexceptionally if they travel between OD pair 1, although they may not travel randomly. They always take route 2 when traveling between OD pair 2. Similarly, the drivers in region III may continually select route 2 between OD pair 1 and route 1 between OD pair 2. Thus, each latent driver is *assigned* to one of four regions: I, II, III, and IV, and these assignment shares are $r_{11}r_{21}$, $r_{11}r_{22}$, $r_{12}r_{21}$, and $r_{12}r_{22}$, respectively. Note that this assignment is endogenously determined through the equilibrium mechanism. The flow Y_{11} on route 1 between OD pair 1 is the number of drivers who make trips in regions I and II, and Y_{12} is the flow for regions III and IV. Each driver makes a trip independently, exogenously, and randomly. Therefore, Y_{11} and Y_{12} are independent when the route choice proportions are given, and, similarly, the route flows between the same OD pair are mutually independent. However, a route flow is not necessarily independent of the route flow between the different OD pairs. The drivers in region I randomly select their OD pair. Therefore, one day they may travel between OD pair 1 and, on another day, travel between OD pair 2, but they take route 1 for both OD pairs. Thus, the flow on route 1 between OD pair 1 is not independent of the flow on route 1 between OD pair 2, because $N_{1k}, N_{2k}, \ldots, N_{Ik}$ are generally correlated. The covariance of flows on route 1 between OD pair 1 and OD pair 2 is $r_{11}r_{21}\sigma_{12}$ (= $0.12\sigma_{12}$ in this case).

The preceding paragraph discusses a simple 2-OD-4-route network. In general, $\mathbf{Y} \sim \mathbb{N}[\mathbf{m}, \mathbf{S}]$. In the case of all uninformed drivers, the mean vector and variance-covariance matrix of route flows are

$$
\mathbf{m} = \begin{pmatrix} r_{11}\mu_1 \\ r_{12}\mu_1 \\ \vdots \\ r_{21}\mu_2 \\ \vdots \\ r_{IJ_I}\mu_I \end{pmatrix} \tag{6}
$$

and

$$
\mathbf{S} = \begin{pmatrix}
r_{11}\sigma_1^2 & & \mathbf{0} & r_{11}r_{21}\sigma_{12} & \cdots & r_{11}r_{2J_1}\sigma_{12} & \cdots \\
 & \ddots & & \vdots & \ddots & \vdots & \\
\mathbf{0} & & r_{1J_1}\sigma_1^2 & r_{1J_1}r_{21}\sigma_{12} & \cdots & r_{1J_1}r_{2J_1}\sigma_{12} & \\
r_{11}r_{21}\sigma_{12} & \cdots & r_{1J_1}r_{21}\sigma_{12} & r_{21}\sigma_2^2 & & \mathbf{0} & \\
\vdots & \ddots & \vdots & & \ddots & & \\
r_{11}r_{2J_1}\sigma_{12} & \cdots & r_{1J_1}r_{2J_1}\sigma_{12} & \mathbf{0} & & r_{2J_1}\sigma_2^2 & \\
\vdots & & & & & & \ddots
\end{pmatrix}. \tag{7}
$$

The link flow distributions are also given using the above multinormally distributed route flows. Clearly,

$$
X_a = \sum_{i=1}^{I}\sum_{j=1}^{J_i} \delta_{a,ij} Y_{ij} \,. \tag{8}
$$

The link flow vector is $\mathbf{X} = \mathbf{\Delta Y}$. Because of the property of multivariate normal distributions (Stuart & Ord, 1994, p. 512), the link flow vector $\mathbf{X}$ follows a multivariate normal distribution

$$
\mathbf{X} \sim \mathbb{N}[\mathbf{\Delta m}, \mathbf{\Delta S \Delta^T}] \,. \tag{9}
$$

Informed drivers choose routes based on the provided travel times. Each day, travel time information is provided and on that day, informed drivers choose their routes deterministically. However, the actual travel times fluctuate daily because travel demand is random, so their route choice proportions also fluctuate from day to day. Thus, route choice proportions of informed drivers are random day to day even if they are decided deterministically on each day. If all drivers are informed, $Y_{ij} = P_{ij} Q_i$ (i = 1, 2,…, I). Unlike the case for uninformed drivers, $P_{ij} Q_i$ is not necessarily normally distributed, as P_{ij} and Q_i are both random. Even so, it can approximately be normally distributed, due to the central limit theorem, when the demand is sufficiently large.

With both informed and uninformed drivers, the random route flow is $Y_{ij} = [(1-\pi)r_{ij} + \pi P_{ij}]Q_i$ (i = 1, 2,…, I). We can treat the route choice proportion r_{ij} of uninformed drivers as a constant once it is determined. However, the route choice proportion P_{ij} of informed drivers is random. Moreover, it is difficult to derive the probability distribution of P_{ij}. In this study, we have to estimate P_{ij} by simulation.

4. Mean and Variance-Covariance of Travel Times

The case in which all drivers are uninformed is a benchmark when assessing the effect of providing information. We call this the "no-information case." In the no-information case, route flows are normally distributed, as described in the previous section. However, it is difficult to derive a tractable probability distribution of route travel times, even if the route flows are normally distributed. In this study, we do not directly manipulate the route travel time distributions. Instead, we use the means and variance-covariance of route travel times.

The mean travel time on link a is

$$\mathrm{E}[T_a] = \mathrm{E}[t_a(X_a)] = \int_{-\infty}^{\infty} t_a(x) f_{X_a}(x)\,dx \ . \tag{10}$$

The probability density function of the normal distribution is not analytically integrable, so we must use a numerical integral or another method with a high computational cost.

We adopt a BPR-type travel time function for calculating travel time, $t_a(x_a) = \tau_a\,[1+\alpha(x_a/\gamma_a)^\beta]$. For simplicity, we express link travel times as $\tau_a + \xi_a x_a^{\ \beta}$, where $\xi_a = \alpha\,\tau_a\,\gamma_a^{-\beta}$. When β is an integer (typically 4 is used), the mean link travel time can be calculated using the characteristic function. Unless the computational cost is questioned, any other approach can be applied to model the travel cost or utility under uncertain conditions.

The characteristic function $\phi(z)$ is defined as $\mathrm{E}[e^{\mathrm{i}zX}]$ (Stuart & Ord, 1994; Papoulis, 1965). As a property of the characteristic function, $\mathrm{E}[X^\beta] = (-\mathrm{i})^\beta d^\beta\phi(z)/dz^\beta|_{z=0}$ (Stuart & Ord, 1994, Eq. (3.18); Papoulis, 1965, p. 157). The mean travel time over link a is

$$\mathrm{E}[T_a] = \tau_a + (-\mathrm{i})^\beta \xi_a \left.\frac{d^\beta \phi_a(z)}{dz^\beta}\right|_{z=0} , \tag{11}$$

where $\phi_a(z)$ is the characteristic function of X_a. The variance of the link travel time is $\mathrm{Var}[T_a] = \mathrm{E}[T_a^{\ 2}] - \{\mathrm{E}[T_a]\}^2$, where $\mathrm{E}[T_a^{\ 2}]$ is also calculated using characteristic functions.

The variances and covariance of X_a and $X_{a'}$ are given by Eq. (9). Even if the number of routes is greater than two, the bivariate normal distribution of X_a and $X_{a'}$ can be given as a marginal distribution of $\mathbf{X}$. Because they follow the bivariate normal distribution, the characteristic function $\phi_{a,a'}(z_a, z_{a'})$ of X_a and $X_{a'}$ is

$$\phi_{a,a'}(z_a, z_{a'}) = \exp\left[(\tilde{m}_a, \tilde{m}_{a'}) \begin{pmatrix} z_a \\ z_{a'} \end{pmatrix} + \frac{1}{2} (z_a, z_{a'}) \begin{pmatrix} \tilde{s}_a^2 & \tilde{s}_{a,a'} \\ \tilde{s}_{a,a'} & \tilde{s}_{a'}^2 \end{pmatrix} \begin{pmatrix} z_a \\ z_{a'} \end{pmatrix} \right]. \tag{12}$$

The variance of the route travel time can be calculated using

$$\mathrm{E}[X_a^{\ \alpha} X_{a'}^{\ \beta}] = (-\mathrm{i})^{\alpha+\beta} \left.\frac{\partial^{\alpha+\beta} \phi_{a,a'}(z_a, z_{a'})}{\partial z_a^{\ \alpha}\, \partial z_{a'}^{\ \beta}}\right|_{z_a=0,\ z_{a'}=0} . \tag{13}$$

The variance of travel time on route j between OD pair i is

$$\omega_{ij}^{\ 2} = \sum_{a=1}^{A} \sum_{a'=1}^{A} \delta_{a,ij}\, \delta_{a',ij}\, \mathrm{Cov}[T_a, T_{a'}], \tag{14}$$

where $\mathrm{Cov}[T_a, T_{a'}] = \mathrm{Var}[T_a]$ if $a' = a$. To obtain the variance or SD of route travel times, we have to calculate the covariance of link travel times because $\mathrm{Cov}[T_a, T_{a'}] = \mathrm{E}[T_a T_{a'}] - \mathrm{E}[T_a]\mathrm{E}[T_{a'}] = \mathrm{E}[(\tau_a + \xi_a X_a^{\beta})(\tau_{a'} + \xi_{a'} X_{a'}^{\beta})] = \tau_a \tau_{a'} + \tau_{a'} \xi_a \mathrm{E}[X_a^{\beta}] + \tau_a \xi_{a'} \mathrm{E}[X_{a'}^{\beta}] + \xi_a \xi_{a'} \mathrm{E}[X_a^{\beta} X_{a'}^{\beta}] - \mathrm{E}[T_a]\mathrm{E}[T_{a'}]$,

$$\begin{aligned}\mathrm{Cov}[T_a, T_{a'}] = {} & (-\mathrm{i})^{\beta}(\tau_{a'} - 1)\xi_a \left.\frac{d^{\beta}\phi_a(z)}{dz^{\beta}}\right|_{z=0} + (-\mathrm{i})^{\beta}(\tau_a - 1)\xi_{a'} \left.\frac{d^{\beta}\phi_{a'}(z)}{dz^{\beta}}\right|_{z=0} \\ & + (-\mathrm{i})^{2\beta}\xi_a \xi_{a'} \left.\frac{\partial^{2\beta}\phi_{a,a'}(z_a, z_{a'})}{\partial z_a{}^{\beta}\,\partial z_{a'}{}^{\beta}}\right|_{z_a=0,\, z_{a'}=0} + \tau_a \tau_{a'} - \tau_a - \tau_{a'}\end{aligned}, \quad (15)$$

using Eqs. (11) and (13). Thus, the covariance $\mathrm{Cov}[T_a, T_{a'}]$ of travel times over links a and a' can also be calculated using characteristic functions.

5. Formulation

5.1. Network equilibrium for the no-information case

The link travel time is $T_a = t_a(X_a)$. As described in Section 2, the efficient travel time $c_{ij} = \bar{t}_{ij} + \eta\,\omega_{ij}$ is used to choose a route in the no-information case. The means and SDs of travel times are calculated using the method proposed in the previous section, where the mean vector and variance-covariance matrix of route flows, **m** and **S**, are given. Because the demand distribution, $\mathbb{N}[\boldsymbol{\mu}, \boldsymbol{\Sigma}]$, is given, **m** and **S** are both functions of **r**, as show in Eqs. (6) and (7). Thus, the efficient travel time function c_{ij} is a function of **r** in the no-information case. In the case with information, the efficient travel time function is a function of **r** and **P** that will be described later.

As Assumption A4 states, the route choice proportion **r** is given by the random utility discrete choice model. Let $h_{ij}(\mathbf{c})$ denote the random utility discrete choice function. This function gives the proportion at which route j between OD pair i is chosen. When the multinomial logit model is adopted, $h_{ij}(\mathbf{c}) = \exp(-\theta c_{ij}) / \Sigma_{j'=1}^{J_i} \exp(-\theta c_{ij'})$. In this case, θ is an exogenous constant, which means the error terms follow an independent and identical Gumbel distribution that represents unobserved factors. As stated above, the vector of efficient route travel times, **c**, is flow-dependent, and is a vector function of **Y**. In the no-information case, $\mathbf{Y} = \mathrm{diag}(\mathbf{Q})\,\mathbf{r}$, and the efficient travel time vector function is expressed by $\mathbf{c}[\mathrm{diag}(\mathbf{Q})\,\mathbf{r}]$. Then, the stochastic network equilibrium model can be formulated as a fixed-point problem:

$$\mathbf{r} = \mathbf{h}\{\mathbf{c}[\mathrm{diag}(\mathbf{Q})\,\mathbf{r}]\}. \quad (16)$$

where $\mathbf{h}(\mathbf{c}) = (h_{11}(\mathbf{c}), \ldots, h_{IJ_I}(\mathbf{c}))^{\mathrm{T}}$. Equation (16) can be reformulated as the following complementary problem.

$$\begin{aligned} & \text{Find } \begin{bmatrix} \mathbf{r} \\ \boldsymbol{\varsigma} \end{bmatrix} \in R_+^J \times R_+^I \\ & \quad \text{such that } \left\langle (\mathbf{r}, \boldsymbol{\varsigma})^{\mathrm{T}}, \boldsymbol{\psi}(\mathbf{r}, \boldsymbol{\varsigma}) \right\rangle = 0,\ \boldsymbol{\psi}(\mathbf{r}, \boldsymbol{\varsigma}) \geq \mathbf{0},\ \mathbf{r} \geq \mathbf{0},\ \boldsymbol{\varsigma} \geq \mathbf{0} \end{aligned} \quad (17)$$

where

$$\boldsymbol{\psi}(\mathbf{r},\varsigma)=\begin{bmatrix}\mathbf{0} & -\boldsymbol{\Gamma}^{\mathrm{T}}\\ \boldsymbol{\Gamma} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{r}\\ \varsigma\end{bmatrix}+\begin{bmatrix}\mathbf{c}[\mathrm{diag}(\mathbf{Q})\mathbf{r}]+\mathbf{ln}(\mathbf{r})/\theta\\ -\mathbf{I}\end{bmatrix}. \tag{18}$$

5.2. Network equilibrium for the information case

As described above, uninformed drivers choose a route based on the efficient travel times of routes. The traffic state is assumed to be stationary and equilibrated, and the efficient travel times remain deterministic. Therefore, the route choice proportions for uninformed drivers are also deterministic.

Informed drivers choose routes based on travel times provided by the road manager on each day. Because the actual route travel times fluctuate daily, their route choice proportions also fluctuate daily. Thus, the route choice proportions for informed drivers are random and are deterministic for uninformed drivers (over a time span of several days). Accordingly, the formulation of an equilibrium with informed drivers differs from that of the no-information case.

Informed drivers possess a priori knowledge of the exact travel time, so they do not need to incorporate a safety margin. As Assumption A4 states, the route choice proportions are given deterministically on each day by the random utility discrete choice model. First, we consider the case in which all drivers are informed. In this case, with the route choice determined by the random utility model, the deterministic network equilibrium is reached when travel demand for the day is given. The network equilibrium on day l is formulated as follows:

$$\mathbf{p}_l=\mathbf{h}\big(\mathbf{t}[\mathrm{diag}(\mathbf{q}_l)\mathbf{p}_l]\big). \tag{19}$$

As mentioned above, travel demand is random and fluctuates daily. The route choice proportions for informed drivers and route flows are also random, even if the deterministic network equilibrium with route choice determined by the random utility model is reached each day. Therefore, using the random variables, Eq. (19) can also be expressed as $\mathbf{P}=\mathbf{h}\big(\mathbf{t}[\mathrm{diag}(\mathbf{Q})\mathbf{P}]\big)$.

The demand $\mathbf{Q}$ is multinormally distributed. Because it is difficult to analytically derive the distribution $\mathbf{P}$ of the random vector of route choice proportions, we are obliged to use a simulation to solve the problem of the stochastic network equilibrium with informed drivers. A simple simulation method is used to randomly generate travel demand and iteratively calculate the deterministic network assignment.

The number of informed drivers between OD pair i is πQ_i. If informed and uninformed drivers coexist, the stochastic network equilibrium requires finding $(\mathbf{r},\mathbf{P})^{\mathrm{T}}$ such that

$$\begin{cases}\mathbf{r}=\mathbf{h}\big(\mathbf{c}\big[\mathrm{diag}(\mathbf{Q})\{(1-\pi)\mathbf{r}+\pi\mathbf{P}\}\big]\big)\\ \mathbf{P}=\mathbf{h}\big(\mathbf{t}\big[\mathrm{diag}(\mathbf{Q})\{(1-\pi)\mathbf{r}+\pi\mathbf{P}\}\big]\big)\end{cases}, \tag{20}$$

because $\mathbf{Y}=\mathrm{diag}(\mathbf{Q})\{(1-\pi)\mathbf{r}+\pi\mathbf{P}\}$ in the information case.

5.3. Assessment of providing information

In the no-information case, all drivers take the efficient travel time to reach their destination. In the information case, informed drivers simply depart at the desired arrival time minus the provided exact travel time and they consume the route travel time. Even in the information case, the uninformed drivers spend the efficient travel time. The travel time fluctuates daily, as stated before. In contrast, the efficient travel time is deterministic, even if the travel times oscillate. The mean time savings for the exact travel time information provision is

$$\sum_{i=1}^{I}\mu_i\sum_{j=1}^{J_i} r_{ij}^o c_{ij}^o - \left(\sum_{i=1}^{I}[1-\pi_i]\mu_i\sum_{j=1}^{J_i} r_{ij}^w c_{ij}^w + \sum_{i=1}^{I}\pi_i\sum_{j=1}^{J_i}\mathrm{E}[Q_i P_{ij} T_{ij}^w]\right), \tag{21}$$

where r_{ij}^w and r_{ij}^o are the route choice proportions on route j between OD pair i in the information and no-information cases, c_{ij}^w and c_{ij}^o are the efficient travel times on route j between OD pair i in the information and no-information cases, respectively, and T_{ij}^w is the random travel time on route j between OD pair i in the information case. Note that the information provision does not necessarily give positive effect, as Huang et al. (2011) and others have discussed.

The total travel time $\sum_{i=1}^{I}\sum_{j=1}^{J_i} Q_i P_{ij} T_{ij}^w$ can be obtained by simulation as mentioned above. An approximate method of calculating $\mathrm{E}[\sum_{i=1}^{I}\sum_{j=1}^{J_i} Q_i P_{ij} T_{ij}^w]$ is also proposed briefly as follows. When the actual demands are given, the route choice proportions and travel times, in the case that drivers are all informed, are given by the standard SUE. Therefore, the total travel time is a function of demands, such that $g(\mathbf{q})$ is the total travel time function in the case that all drivers are informed. The total travel time function is not generally explicit, so an approximate method is examined. The Taylor expansion around the mean demand $\boldsymbol{\mu}$ yields $g(\mathbf{Q}) \approx g(\boldsymbol{\mu}) + \nabla_{\mathbf{q}} g(\mathbf{q})\big|_{\mathbf{q}=\boldsymbol{\mu}}(\mathbf{Q}-\boldsymbol{\mu})$ as a first-order approximate. Then, $\mathrm{E}[g(\mathbf{Q})] \approx g(\boldsymbol{\mu}) + \nabla_{\mathbf{q}} g(\mathbf{q})\big|_{\mathbf{q}=\boldsymbol{\mu}}\mathrm{E}[\mathbf{Q}-\boldsymbol{\mu}] = g(\boldsymbol{\mu})$ because $\boldsymbol{\mu} = \mathrm{E}[\mathbf{Q}]$. Thus, the first-order approximate mean total travel time is $g(\boldsymbol{\mu})$, that is, the total travel time of the standard SUE with mean demands.

Equation (21) allows us to calculate the savings in travel cost. In some cases, satisfaction (Daganzo, 1979, p. 128) could be more appropriate to evaluate a transportation policy when the route choice is made according to the random utility discrete choice model. If the route choice is made as per the multinomial logit model, the satisfaction is modeled as a log sum. In this case, the benefit of providing the exact travel time information is

$$-\frac{1}{\theta}\left\{\sum_{i=1}^{I}\mu_i \ln\sum_{j=1}^{J_i}\exp\left[-\theta c_{ij}^o\right] - (1-\pi)\sum_{i=1}^{I}\mu_i \ln\sum_{j=1}^{J_i}\exp(-\theta c_{ij}^w) - \pi\,\mathrm{E}\left[\sum_{i=1}^{I} Q_i \ln\sum_{j=1}^{J_i}\exp(-\theta T_{ij}^w)\right]\right\} \tag{22}$$

Next, the case that a minority of drivers are informed is discussed. We will examine the benefit of providing such information to ambulances in Kanazawa, Japan, in the next section. In this case, the number of uninformed drivers is much larger than that of the informed drivers, so the stochastic network equilibrium in the no-information case can be assumed as a whole traffic state, because the effect of a small number of informed drivers to traffic is negligible. The ambulances must reach their destinations as soon as possible, so they should choose the route with the minimum travel time. Without travel time information, they choose the route with the minimum *mean* travel time, which is determined by the travel time distributions. They take the same route between the same OD pair every day even if the travel time on this route is random. Therefore, their mean travel time is $\min[\bar{t}_{ij} \mid j=1,2,...,J_i]$. However, the route with the minimum *mean* travel time may differ from the route with the minimum *actual* travel time.

The informed drivers are able to take the route with the *actual* minimum travel time on each trip. The series of their experienced route travel times are $\{\min[t_{ij1}], \min[t_{ij2}], \min[t_{ij3}],...\}$. Thus, their travel time follows a type of extreme value distribution of random route travel times: $\min[T_{ij} \mid j=1,2,...,J_i]$. In this case, the savings in travel time for informed drivers is

$$\min[\bar{t}_{ij} \mid j=1,2,...,J_i] - \mathrm{E}\left(\min[T_{ij} \mid j=1,2,...,J_i]\right). \tag{23}$$

6. Example

6.1 Simple network

To illustrate the above proposed methods, we present a simple example of a network consisting of two OD pairs and three links. Such a network is one of the simplest networks with multiple OD pairs, links, and routes. Figure 2 shows the network, which has four routes. Route 1 between OD pair 1 consists of link 1 and link 2 and route 2 between OD pair 1 is comprised of link 1 and link 3. The link route incidence matrix for this network is

$$\mathbf{\Delta} = \begin{pmatrix} \delta_{1,11} & \delta_{1,12} & \delta_{1,21} & \delta_{1,22} \\ \delta_{2,11} & \delta_{2,12} & \delta_{2,21} & \delta_{2,22} \\ \delta_{3,11} & \delta_{3,12} & \delta_{3,21} & \delta_{3,22} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}. \tag{24}$$

The normally distributed OD demands are as follows.

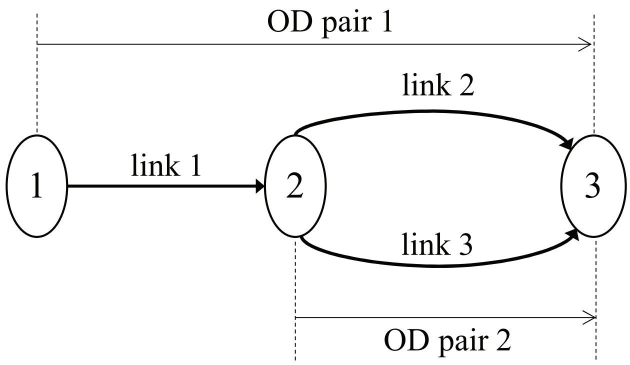

Fig. 2. A simple network.

Table 1. Capacities and free-flow travel times of link travel-time functions.

	Free-flow travel time	Capacity
Link 1	10	20
Link 2	10	20
Link 3	5	10

$$\begin{pmatrix} Q_0 \\ Q_1 \\ Q_2 \end{pmatrix} \sim \mathbb{N}\left[\begin{pmatrix} 10 \\ 16 \\ 9 \end{pmatrix}, \begin{pmatrix} 8 & 2 & 2 \\ 2 & 9 & 2 \\ 2 & 2 & 6 \end{pmatrix}\right]. \tag{25}$$

The multinomial logit model is adopted to determine discrete choices of routes. We set the diversion parameter θ in the logit model to 0.1 and the risk attitude parameter η to 1.0. The route choice proportions for uninformed drivers are $r_{ij} = \exp[-(\bar{t}_{ij} + \omega_{ij})] / \sum_{j'=1}^{J_i} \exp[-(\bar{t}_{ij'} + \omega_{ij'})]$. The standard (or deterministic) link travel time function is $t_a(x_a) = \tau_a[1 + (x_a/\gamma_a)^2]$, and capacities and free flow travel times of the three links in the network are given in Table 1.

6.1.1 No-Information Case

The mean and variance of link travel times were derived using the method in Section 4. In the no-information case of the example, the mean of the link travel time is expressed as

$$\tau_a\left[1 + \frac{m_a^2 + s_a^2}{\gamma_a^2}\right]. \tag{26}$$

The variance is given by

$$\frac{2\tau_a{}^2\left[2m_a^2 s_a^2 + s_a^4\right]}{\gamma_a^4}. \tag{27}$$

In this section, r_1 and r_2 are used instead of r_{11} and r_{21} for simpler expression. Therefore, the proportion of choosing route 2 between OD pair 1 is $1-r_1$, while $\bar{t}_{11} - \bar{t}_{12} = \bar{t}_{21} - \bar{t}_{22}$, $c_{11} - c_{12} \neq c_{21} - c_{22}$. Therefore, $r_1 \neq r_2$. Then, according to Eqs. (6) and (7), the mean vector and variance-covariance matrix of route flows are

$$\mathbf{m} = \begin{pmatrix} m_{11} \\ m_{12} \\ m_{21} \\ m_{22} \end{pmatrix} = \begin{pmatrix} 16r_1 \\ 16(1-r_1) \\ 9r_2 \\ 9(1-r_2) \end{pmatrix} \tag{28}$$

and

$$
\mathbf{S} = \begin{pmatrix} s_{11}^2 & s_{11,12} & s_{11,21} & s_{11,22} \\ s_{11,12} & s_{12}^2 & s_{12,21} & s_{12,22} \\ s_{11,21} & s_{12,21} & s_{21}^2 & s_{21,22} \\ s_{11,22} & s_{12,22} & s_{21,22} & s_{22}^2 \end{pmatrix}
= \begin{pmatrix} 9r_1 & 0 & 2r_1r_2 & 2r_1(1-r_2) \\ 0 & 9(1-r_1) & 2(1-r_1)r_2 & 2(1-r_1)(1-r_2) \\ 2r_1r_2 & 2(1-r_1)r_2 & 6r_2 & 0 \\ 2r_1(1-r_2) & 2(1-r_1)(1-r_2) & 0 & 6(1-r_2) \end{pmatrix}. \tag{29}
$$

The mean vector and variance-covariance matrix of link flows are given by

$$
\begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix} = \begin{pmatrix} 16 \\ 16r_1 + 9r_2 \\ 25 - 16r_1 - 9r_2 \end{pmatrix} \tag{30}
$$

and

$$
\begin{pmatrix} s_1^2 & s_{1,2} & s_{1,3} \\ s_{1,2} & s_2^2 & s_{2,3} \\ s_{1,3} & s_{2,3} & s_3^2 \end{pmatrix} = \begin{pmatrix} 9 & 9r_1 + 2r_2 & 11 - 9r_1 - 2r_2 \\ 9r_1 + 2r_2 & 9r_1 + 6r_2 + 4r_1r_2 & 2(r_1 + r_2 - 2r_1r_2) \\ 11 - 9r_1 - 2r_2 & 2(r_1 + r_2 - 2r_1r_2) & 19 - 13r_1 - 10r_2 + 4r_1r_2 \end{pmatrix}. \tag{31}
$$

Substituting Eqs. (30) and (31) into Eqs. (26) and (27) yields the means and variances of link travel times, and the efficient travel times are obtained. The covariance of link travel times is also obtained.

We obtain $r_1 = 0.504$ and $r_2 = 0.512$ by solving the stochastic network problem of Eq. (16). Table 2 shows the equilibrium link travel times for the example network in the no-information case. Links 1 and 2 share the same attributes. The (mean) travel time of link 1 is greater than that of link 2, and the SD and variance for link 1 are greater than those for link 2. The capacity of link 3 is half that of links 1 and 2, and the flow through link 3 fluctuates more than links 1 and 2. Thus, the SD and variance of link 3 are greater than those of links 1 and 2, although the mean travel time through link 3 is the smallest. We can also calculate the covariance between link travel times, route variances, and

Table 2. Equilibrium travel times in the simple network

	Link 1	Link 2	Link 3
Mean	16.23	14.23	13.01
S.D.	2.42	1.89	3.61
Variance	5.86	3.57	13.04

	Link 1,2	Link 1, 3
Covariance	1.03	1.98

efficient travel times, using Eqs. (11), (14) and (15), respectively. This allows us to evaluate the network uncertainty as the sum of safety margins.

The case that all drivers are informed is also considered. As mentioned in Section 5.3, the first-order approximate of mean total travel time is given by the (deterministic) total travel time of the standard SUE with mean demands. The mean total travel time is approximately 598.78 in the case that all drivers are informed. In contrast, the total efficient travel time is 696.60 in the no-information case. Therefore, the benefit of providing the travel time information to all drivers is a time savings of 97.82. The total of mean demands is 25.0, and 3.91 min. per driver is saved.

6.1.2 Information Case

In the previous sub-section, to evaluate the effect of the information provision, the no-information case is compared with the case where all drivers are informed. The information case where informed and uninformed drivers coexist is much more complicated. It is difficult to solve Eq. (20) analytically, and it is calculated by simulation. Using 1,000 sets of simulated demands, Eq. (20) is numerically solved.

A simple network case, where the proportion of informed latent drivers for all latent drivers is 0.2 in each OD pair, $\pi = 0.2$, is examined in this sub-section. At the (stationary) equilibrium, $r_{11} = 0.503$ ($=r_1$) and $r_{21} = 0.505$ ($=r_2$) for the route choice proportions of uninformed drivers. While the proportion of the uninformed drivers that choose route 1 between OD pair 1 is fixed at 0.503, informed drivers could change the routes each time, even at the stationary equilibrium. The exact travel times are provided to the informed drivers, but they fluctuate because the demands are normally distributed. Therefore, the route choice proportions of informed drivers are stochastic, because they choose the routes based on the *realized* travel time information that oscillates every time.

The mean travel time of informed drivers between OD pair 1 for 1,000 days is as follows: $(1/1000)\Sigma_{l=1}^{1000}(p_{11l}\, t_{11l} + p_{12l}\, t_{12l}) = 30.27$. In contrast, the mean travel time of uninformed drivers between OD pair 1 is $30.32 = r_{11}\Sigma_{l=1}^{1000} t_{11l}/1000 + r_{12}\Sigma_{l=1}^{1000} t_{12l}/1000$. The mean travel time of informed drivers between OD pair 2 is 13.57 and that of uninformed drivers is 13.63. Thus, the mean travel times of informed drivers are less than those of uninformed drivers. The total consumed time, which consists of the total travel time of informed drivers and the total efficient travel time of uninformed drivers, is 592.67 in this case. Compared to the no-information case in the previous sub-section, 4.16 min. per driver is saved, on average, for providing information to 20% of drivers in the example network.

6.2 Information Provision to Ambulances in Kanazawa

Figure 3 shows the road network of central Kanazawa, Japan. The network consists of 140 nodes and 467 links. There are four fire stations with ambulances: Ekinishi, Hirosaka, Chuo, and Naruwa. In Japan, emergency hospitals are classified into first,

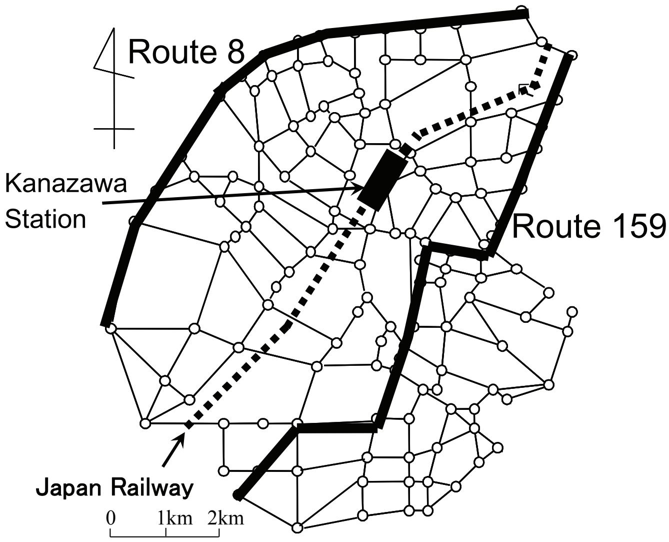

Fig. 3. Kanazawa road network.

second, and third ranks. First-order emergency hospitals provide medical services without surgical operations. Second-order hospitals accept patients requiring operations. Patients with more serious ailments are transferred to third-order emergency hospitals. There are two third-order emergency hospitals in the network. One third-order emergency hospital (Kanazawa University Hospital) is located in the Hirosaka district and the other (Ishikawa Prefecture Hospital) is in the Ekinishi district. Most patients with serious ailments, such as myocardial infarction or multiple traumas, are transported to one of the two third-order emergency hospitals.

The mean travel demands are derived from a personal trip survey within the Kanazawa urban area that was conducted previously. It is difficult to obtain the variance-covariance matrix of travel demands. For simplicity, independent demands are assumed in this case. We observed several links on the network, and found that $s_a^2 = 42 m_a$ represents the relationship between the mean and variance of link flows. Because of independent route flows, $s_{ij}^2 = 42 m_{ij}$ is consistent with $s_a^2 = 42 m_a$, and $\sigma_i^2 = 42\mu_i$ is derived due to independent demands.

In this section, the effect of providing exact travel time information to ambulances is discussed. No other vehicles are informed. Without information, the stochastic network equilibrium expressed by Eq. (16) is established. Unfortunately, we have no information on the risk attitude of drivers in Kanazawa, so we tentatively set the risk parameter η to 0, meaning the uninformed drivers choose routes based on mean route travel times. Estimation of the risk attitude parameter for the stochastic network equilibrium should be a topic of future investigation. Without the travel time information, ambulances and other drivers choose the minimum *mean* travel time routes.

In this example, the multinomial logit model is adopted, but the parameter θ in the logit model is assumed to be sufficiently large. This implies that the drivers have exact knowledge of travel time distributions without perceptual error. Then, the stochastic network equilibrium can be formulated as a link-based optimization problem. This reduces computational cost drastically, and we are freed from generating the set of routes, which includes the overlapping problem. The travel times without the information are obtained by solving the following problem:

$$\min. Z = \sum_{a=1}^{A} \int_0^{m_a} \bar{t}_a(w)\, dw \tag{32}$$

such that

$$\mu_i = \sum_{j=1}^{J_i} m_{ij} \quad , \tag{33}$$

$$m_a = \sum_{i=1}^{I} \sum_{j=1}^{J_i} \delta_{a,ij}\, m_{ij} \quad , \tag{34}$$

$$m_{ij} \geq 0 \, , \tag{35}$$

where $\bar{t}_a(m_a)$ is the mean travel time function, which is derived by the method described in Section 4. We can apply the standard Wardrop user equilibrium algorithms, such as the Frank–Wolfe method.

If only ambulance drivers are provided with exact travel time information, the above stochastic network equilibrium is still established because the effect of ambulances on traffic is negligible. In this section, we focus on the third-order emergency patients, so the destination of ambulances is one of the two third-order emergency hospitals. The travel times fluctuate daily, but the ambulance drivers choose the route with the *actual* minimum travel time each time. The travel time from the fire station to the third-order emergency hospital via a patient's location forms an extreme-value distribution of route travel times. In this section, the mean travel time of ambulances with travel time information is calculated by numerical simulation. Specifically, it is given by

$$\frac{1}{1000} \sum_{l=1}^{1000} \min[t_{ijl} \mid j = 1,2,\ldots,J_i] \, . \tag{36}$$

Thus, the minimum travel time for a given route is obtained by summing over 1000 traffic time assignments.

Results of the numerical calculation of Eq. (36) are summarized in Table 3. The table shows the mean decrease in ambulance transport time for patients at each node in the district. Approximately three minutes are saved by providing ambulance drivers with exact travel time information. The mean of each ambulance transport time is 14.11 min. without the information, and the accurate travel time information provision results in a 21.0% reduction in ambulance transport time in Kanazawa. Furthermore, the variance in travel time is also reduced. Figure 4 gives the reduction in travel time for

Table 3. Reduction of mean and SD of ambulance travel time.

	travel time reduction (min.)	reduction of SD of travel time
Ekinishi	2.74	2.23
Hirosaka	0.23	0.49
Chuo	3.46	1.96
Naruwa	5.46	3.25
Average	2.97	1.99

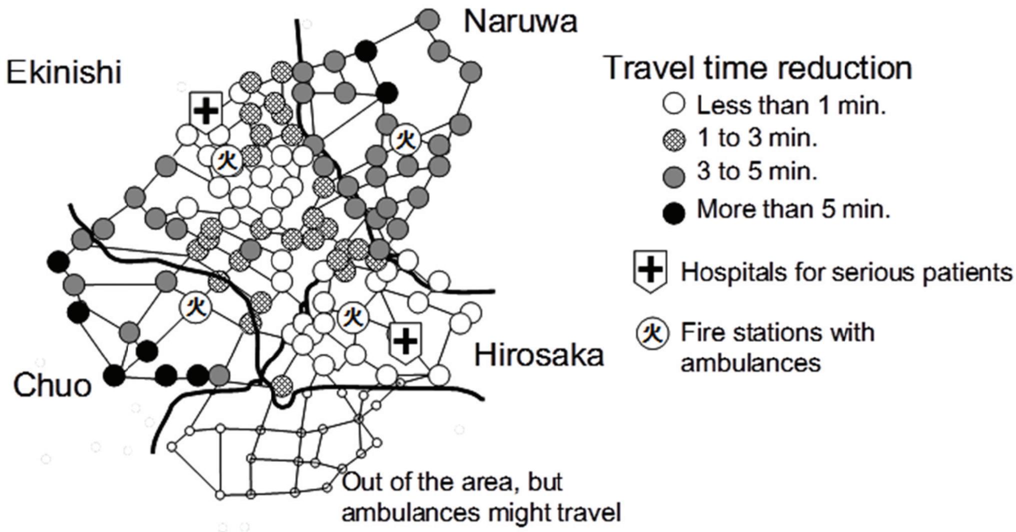

Fig. 4. Reduction in travel time from each node to a given hospital.

each node in the network. The reduction in travel time for patients who reside far from the third-order emergency hospital is larger than that for patients who live closer. These results demonstrate that the proposed method makes it possible to determine the effect of providing emergency vehicles with precise traffic information.

7. Conclusions

In this study, to estimate travel times through road networks, we assume stochastic demands and propose a stochastic network equilibrium model whose travel times, flows, and demands are stochastic. Evaluating uncertainty in traffic networks is important for assessing systems that provide traffic information. We focus on travel time information and propose a model to assess the effect of providing a priori travel time information. The proposed model enables us to evaluate how travel time is affected by providing this information on the various possible routes a driver can take.

To examine the feasibility and validity of the proposed stochastic network equilibrium model and methods to determine the effects of a priori knowledge of traffic information, we apply them to a simple network and the real road network of Kanazawa, Japan. The results indicate that providing ambulance drivers in Kanazawa with a priori travel time information leads to an average reduction in travel time of approximately three minutes.

When applying these methods to real networks, we must estimate several parameters including a risk attitude parameter and the multivariate distribution of OD demands. Future work in this area should include developing a quantitative estimate of these parameters. The demand distributions are exogenous in this study, but could be given endogenously. In addition, simultaneous origin-destination and route choices should be considered in the future. The proposed model is static, but dynamic aspects may be required in some cases; therefore, dynamic stochastic network equilibrium models should be formulated in the future. The behavioral impacts of traffic information are actually more complex than we assumed, e.g. Tseng et al. (2013), Kusakabe & Nakano (2015). These are also future works.

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