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ICROS-SICE International Joint Conference 2009
August 18-21, 2009, Fukuoka International Congress Center, Japan

Impedance Control for Force - Reflecting Teleoperation with Communication Delays Based on IOS Small Gain Theorem

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Abstract: This paper focuses on a new proposal system input of impedance control that relates to force reflection (FR) scheme of teleoperation with time varying in communication lines. To improve the tracking performance and transparency, the control algorithm uses one more communication channel to transfer the FR information from the master side to the slave side. Using variable damping values, the contact stability is achieved at the time while the slave robot contacts with the environment. In this work, the input-to-output stability (IOS) small gain approach is used to show the overall force-reflecting teleoperation to be input-to-state stable (ISS). Several experimental results show the effectiveness of our proposed algorithm.

Keywords: Bilateral teleoperation, impedance control, force-reflection, communication delays, input-to-state stable.

1. INTRODUCTION

Teleoperation systems allow a person to extend his/her intelligence and manipulation capability to remove place and/or hazardous environments through coordinated control of two robotic arms, i.e., a master hand controller that is used by a human operator, and a slave robot that manipulates the environment. During the last several decades, many different teleoperation systems have been developed with wide applications in different circumstances such as use in outer space, undersea, in nuclear plants, in surgical operations, in vehicle steering, etc. and this field is being pursued by many researchers [1].

In bilateral teleoperation, the master and slave robots are coupled via communication lines, where the position and/or force information are transferred. Communication delays are incurred in the transmission of data between the master and the slave. It is well known that the delay in a closed loop system may destabilize and deteriorate the transparency of the teleoperation system [2], [3].

While accurate tracking is essential for the skillful control of tasks, it is not enough to achieve the good performance on its own, since position is not the only relationship that exists between both robots. In fact at the moment that the slave robot starts its interaction with the environment, reflecting forces appear and arise. If we do not notice, this force can be uncontrolled and can become a danger in several tasks. Consequently, the feedback of the force is very important and extremely useful, and it leads to so-called force reflection (FR) in a master-slave system. The FR scheme tries not only to achieve good tracking during unconstrained motion, but also to convey precise information of the forces between the slave robot and the environment. Therefore the operator can actually feel these forces on the master robot [4], [5], [6].

Up to now, many surveys concern the teleoperation control systems. The impedance control was introduced and improved, such as in [7] - [11]. This control method based computed torque approach was used in [8], and the control objective is to make mimic a passive mechanical tool with a force-reflecting ability. Here, the authors also used variable damping to improve the tracking performance concurrently. The research [9] proposed one control method based impedance control for comparison of some controllers in a 2-DOF master-slave system. A new force reflecting teleoperation methodology with adaptive impedance control was used in [10] to reduce operator energy requirements without sacrificing stability. In addition, to improve the transparency of bilateral teleoperation with communication delays, a force-reflection (FR) scheme was addressed in [11] and the method is PD control. In [7] - [11], tracking performance has been achieved by explicit position feedback/feedforward control.

The force-reflecting teleoperation was introduced in [11]. Here the environment force reflected on the master side can be altered depending on the force applied by the human operator. However this alteration is not felt by the human, so the FR algorithm [11] is not effective for the transparency. On the other hand, since there is only the force reflected from the environment, then the control system of [11] has three channels of communication lines for teleoperation. This work also addresses the stabilizing problem of force reflecting teleoperation in the presence of time varying communication delays.

In this paper, we focus on transparency and tracking performance improvement of teleoperation system by using the impedance control based on some results of [7], [9]. This method proposes using a new system input, which relates to a new proposed FR algorithm under time-varying delays in the communication lines. Beside the reflecting force from environment that transferred to the master side [11], our work proposes to transfer the force exerted by the human to the slave side with a system of four communication channels for teleoperation. Also in this paper, based on the proposal in [8], the damping modulation method that uses distance measurement is used to achieve contact stability with good transparency and tracking performance concurrently. In our opinion, the sensation of the human operator is important; by using this proposal with FR algorithm, the human can feel alteration of the force at the end-effector of the slave robot in contact tasks. In this work, the input-to-output stability (IOS) small gain approach [12] - [15] is used to show the overall FR teleoperation system to be input-to-state stable (ISS), and several experimental results show the effectiveness of our proposed algorithm.

2. PROBLEM FORMULATION

2.1 Dynamics of Teleoperation System

In this paper, we consider a pair of robotic system coupled via communication lines with time-varying delays. Assuming the absence of friction, other disturbances and gravity term, the master and slave dynamics with $n-$ DOF are described as:

$$\begin{cases} M_m(q_m)\ddot{q}_m + C_m(q_m,\dot{q}_m)\dot{q}_m = \tau_m + J_m^T F_{op} \\ M_s(q_s)\ddot{q}_s + C_s(q_s,\dot{q}_s)\dot{q}_s = \tau_s - J_s^T F_e \end{cases} \tag{1}$$

where the subscript "m" and "s" denote the master and slave indexes, respectively, $q_m,\ q_s \in R^{n\times 1}$ are the joint angle vectors, $\dot{q}_m,\ \dot{q}_s \in R^{n\times 1}$ are the joint velocity vectors, $\ddot{q}_m,\ \ddot{q}_s \in R^{n\times 1}$ are the joint acceleration vectors, $\tau_m,\ \tau_s \in R^{n\times 1}$ are the input torque vectors, $F_{op} \in R^{n\times 1}$ is the operational force vector applied to the master robot by the human operator, $F_e \in R^{n\times 1}$ is the environmental force vector applied to the environment by the slave robot, $M_m,\ M_s \in R^{n\times n}$ are the symmetric and positive definite inertial matrices, $C_m\dot{q}_m,\ C_s\dot{q}_s \in R^{n\times 1}$ are the centripetal and Coriolis torque vectors, and $J_m,\ J_s \in R^{n\times n}$ are Jacobian matrices.

Considering that position encoders measure manipulator coordinate q_i, Cartesian coordinate must be related to these coordinate. Their derivatives through the Jacobian matrix $J_i(q_i)$ with $i = m,\ s$ as follows:

$$z_i = h_i(q_i(t)) \Rightarrow \dot{z}_i = J_i(q_i)\dot{q}_i \tag{2}$$

Following the motion of the master, the slave manipulator interacts with the environment. Here the environment is assumed to be a dynamic system described by the equations below in the form of input-to-output stability properties:

$$\begin{cases} \dot{x}_e = F_{env}(x_e, z_s, \dot{z}_s, t) \\ F_e = G_{env}(x_e, z_s, \dot{z}_s, t) \end{cases} \tag{3}$$

where $x_e \in R^z$ is a state of the environment, we assume that $F_{env}(x_e, z_s, \dot{z}_s, t)$ is measurable in t, locally Lipschitz in $x_e,\ z_s,\ \dot{z}_s$ and essentially bounded on any compact set of $x_e,\ z_s,\ \dot{z}_s$ uniformly in $t \geq 0$. Additionally, suppose

$$|G_{env}(x_e, z_s, \dot{z}_s, t)| \leq a(|x_e| + |z_s| + |\dot{z}_s| + |b|) \tag{4}$$

holds for all $t \geq 0$, where $a,\ b \geq 0$. The following assumption is imposed on the environmental dynamics:

Assumption 1: There exists a locally Lipschitz storage function $V_e : R^z \to R$, $\alpha_{1e},\ \alpha_{2e} \in \mathscr{K}_\infty$, and $\alpha_{3e} > 0$ such that: $\alpha_{1e}(|x_e|) \leq V_e(x_e) \leq \alpha_{2e}(|x_e|)$ holds for all $x_e \in R^z$, and the time derivative of V_e along trajectories of (3) satisfies:

$$\dot{V}_e(t) \leq -\alpha_{3e}|x_e|^2 + F_e^T s_e(t)$$

for almost all $t \geq 0$, where:

$$s_e(t) = \dot{z}_s(t) - z_2^*(t) + \Lambda_{env}(z_s(t) - z_1^*(t)) \tag{5}$$

where $\Lambda_{env} = \Lambda_{env}^T \in R^{n\times n}$, and $z_1^*,\ z_2^* : R \to R^n$ are some continuous uniformly bounded functions.

2.2 Control Objectives

We would like to design the control input τ_m and τ_s to achieve a task-space synchronization and the transparency improvement with proposed force reflection algorithm of teleoperation. Let us define the position tracking errors of the end-effector as follows:

$$\begin{cases} e_m(t) = z_m(t) - z_s(t - T_s(t)) \\ e_s(t) = z_s(t) - z_m(t - T_m(t)) \end{cases} \tag{6}$$

where z_m and $z_s \in R^{n\times 1}$ are the end-effector position vectors. The control objectives in this paper as follows:

1. The synchronization is achieved as:

$$e_i(t),\ \dot{e}_i(t) \to 0\ as\ t \to \infty,\ i = m, s \tag{7}$$

2. The transparency is achieved with $\ddot{z}_i(t) = \dot{z}_i(t) = 0$, $i = m,\ s$ as:

$$F_{op}(t) \to -F_e(t) \tag{8}$$

2.3 Impedance Controller

A precise knowledge about the values of the dynamic parameters allows the implementation of an inverse dynamics algorithm as impedance controller. Here, following the proposal in [9], the torque given by the motors can be split into two terms, the first arising from the teleoperation τ_{tel}, and the second from the impedance control τ_{inv}. The torque inputs of the system are as follows:

$$\tau_i = \tau_{inv_i} + \tau_{tel_i} \tag{9}$$

where the second term is defined as: $\tau_{tel\,i} = J_i^T F_{tel\,i}$ $(i = m,\ s)$. If we assume H_i and B_i to be the mass and damping and they are assumed positive definite diagonal matrices; z_i is a vector containing the Cartesian coordinates. $F_{ext\,m/s}$ represents the forces exerted on each robot which include reflection force information in, and $F_{tel\,m/s}$ represents the forces via teleoperation. Applying the approach in [7], the target relationship between the movement of each robot and the force that acts on it is expressed as follows:

$$\begin{cases} H_m\ddot{z}_m + B_m\dot{z}_m = F_{ext\,m} + F_{tel\,m} \\ H_s\ddot{z}_s + B_s\dot{z}_s = F_{tel\,s} + F_{ext\,s} \end{cases} \tag{10}$$

Concerning (2) we get the further differentiation as:

$$\ddot{z}_i = J_i(q_i)\ddot{q}_i(t) + \dot{J}(q_i)\dot{q}_i^2 \tag{11}$$

Substituting (11) and (2) in to (10) and operating, we can calculate the acceleration of the system as follows:

$$\begin{cases} \ddot{q}_m = H_m^{-1}J_m^{-1}[F_{ext\,m} + F_{tel\,m} - B_m J_m \dot{q}_m] - J_m^{-1}\dot{J}_m\dot{q}_m^2 \\ \ddot{q}_s = H_s^{-1}J_s^{-1}[F_{ext\,s} + F_{tel\,s} - B_s J_s \dot{q}_s] - J_s^{-1}\dot{J}_s\dot{q}_s^2 \end{cases} \tag{12}$$

Here for simplicity, we assume that:

Assumption 2: The Jacobian $(J_m,\ J_s)$ are invertible, i.e. they are nonsingular matrices at all times in operation. They are also called pseudoinverse matrices.

Substituting (12) and (9) into (1) and enclosing the above assumption, we get:

$$\begin{cases} \tau_{inv\,m} = M_m H_m^{-1} J_m^{-1}[F_{ext\,m} + F_{tel\,m}] - M_m H_m^{-1} B_m \dot{q}_m \\ \qquad -M_m J_m^{-1}\dot{J}_m \dot{q}_m^2 + C_m\dot{q}_m - (J_m^T F_{tel\,m} + J_m^T F_{op}) \\ \tau_{inv\,s} = M_s H_s^{-1} J_s^{-1}[F_{ext\,s} + F_{tel\,s}] - M_s H_s^{-1} B_s \dot{q}_s \\ \qquad -M_s J_s^{-1}\dot{J}_s \dot{q}_s^2 + C_s\dot{q}_s - (J_s^T F_{tel\,s} - J_m^T F_e) \end{cases} \tag{13}$$

We receive the master slave robot dynamics with impedance controller by substituting (13) into (1) with enclosing (9) as:

$$\begin{cases} M_m\ddot{q}_m + C_m\dot{q}_m = M_m H_m^{-1} J_m^{-1}[F_{ext\,m} + F_{tel\,m}] \\ \qquad -M_m H_m^{-1} B_m \dot{q}_m - M_m J_m^{-1}\dot{J}_m \dot{q}_m^2 + C_m\dot{q}_m \\ M_s\ddot{q}_s + C_s\dot{q}_s = M_s H_s^{-1} J_s^{-1}[F_{ext\,s} + F_{tel\,s}] \\ \qquad -M_s H_s^{-1} B_s \dot{q}_s - M_s J_s^{-1}\dot{J}_s \dot{q}_s^2 + C_s\dot{q}_s \end{cases} \tag{14}$$

From (14) we get:

$$\begin{cases} H_m(\dot{J}_m\dot{q}_m^2 + J_m\ddot{q}_m) + B_m J_m \dot{q}_m = F_{ext\,m} + F_{tel\,m} \\ H_s(\dot{J}_s\dot{q}_s^2 + J_s\ddot{q}_s) + B_s J_s \dot{q}_s = F_{tel\,s} + F_{ext\,s} \end{cases} \tag{15}$$

Considering (2) and (11) we receive the task space dynamics of the teleoperation system as follows:

$$\begin{cases} H_m\ddot{z}_m + B_m\dot{z}_m = F_{ext\,m} + F_{tel\,m} \\ H_s\ddot{z}_s + B_s\dot{z}_s = F_{tel\,s} + F_{ext\,s} \end{cases} \tag{16}$$

In the impedance controller, we propose the exerted forces of each robot on both sides of the teleoperation system, in which reflecting forces are also addressed:

$$\begin{cases} F_{ext\,m}(t) = F_{op}(t) - \tilde{F}_e(t - T_s(t)) \\ F_{ext\,s}(t) = \tilde{F}_{op}(t - T_m(t)) - F_e(t) \end{cases} \tag{17}$$

where $\tilde{F}_{op}(t - T_m(t))$ and $\tilde{F}_e(t - T_s(t))$ are reflecting forces from master and slave sides of teleoperation, reflectively.

We assume $K_m,\ K_s \in R^{n\times n}$ to be positive definite diagonal gain matrices. The controller of the torque arises from teleoperation is proposed as follows:

$$\begin{cases} F_{tel\,m}(t) = K_m[z_s(t - T_s(t)) - z_m(t)] \\ F_{tel\,s}(t) = K_s[z_m(t - T_m(t)) - z_s(t)] \end{cases} \tag{18}$$

where $T_m(t)$ and $T_s(t)$ are time varying delays in the communication lines. Fig. 1 shows a block diagram of the control system with impedance based force-reflection teleoperation, Fig. 2 is a block of master/slave robot dynamics with an impedance controller.

2.4 Communication Delay

Let $T_i : R \to R^+, i \in m,\ s$ be time-dependent time-delay in the forward $(i = m)$ and backward $(i = s)$ communication channels, respectively. If the positions and velocities of the master and slave are transmitted to each side with communication delays $T_{m/s}(\cdot)$, the following signals

$$\hat{z}_m(t) = z_m(t - T_m(t));\ \dot{\hat{z}}_m(t) = \dot{z}_m(t - T_m(t))$$
$$\hat{z}_s(t) = z_s(t - T_s(t));\ \dot{\hat{z}}_s(t) = \dot{z}_s(t - T_s(t)) \tag{19}$$

are available for the controller on both sides of teleoperation.

On the other hand, a contact force due to the environment is measured on the slave side and transmitted back to the master side. Similarly, the force exerted on the master manipulator also is measured and transmitted forward to the slave side, with communication delay $T_{s/m}(\cdot)$, i.e.

$$\begin{cases} \hat{F}_e(t) = \tilde{F}_e(t - T_s(t)) \\ \hat{F}_{op}(t) = \tilde{F}_{op}(t - T_m(t)) \end{cases} \tag{20}$$

Both $T_m(t)$ and $T_s(t)$ are assumed to be time-varying and possibly unbounded.

3. FORCE-REFLECTION SCHEME

In this section, we will consider the FR teleoperation system with communication delays as a system of functional-differential equations (FDE). A state of overall teleoperation system at time $t \in R$ can be chosen as follows:

$$x_t = (z_m^T, \dot{z}_m^T, z_s^T, \dot{z}_s^T, e_m^T, \dot{e}_m^T, e_s^T, \dot{e}_s^T)^T \tag{21}$$

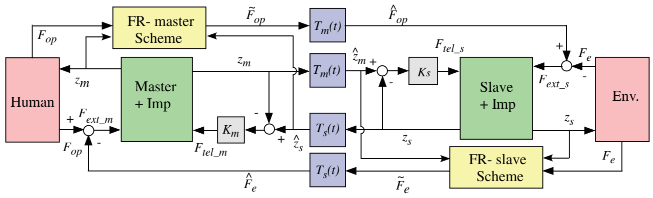

Fig. 1 Teleoperation system.

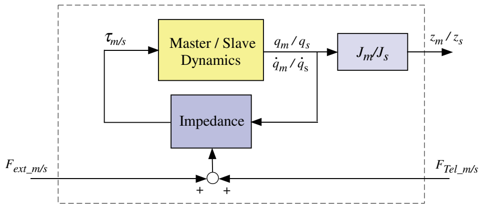

Fig. 2 The master and slave robot dynamics with impedance controller.

The research [11] introduced a new FR algorithm. The reflecting force is transferred from the slave side to the master or operator side. In this strategy, to avoid excessive force pushing against the human operator, the saturation function of FR was used. Note that since the algorithm was proposed to change the FR only when the human operator does not push against the environmental force, this alteration is not felt by the human. Therfore the transparency of the teleoperation system is deteriorated. The simulation results of this strategy did not show the improved transparency of the teleoperation. However, this algorithm may prevent the teleoperator system from going into unstable mode.

In our opinion, the sensation felt by the human operator is important as it allows the human to feel the alteration of the force exerted on the environment by the FR from the slave side. It helps the human to apply an appropriate force in the real task during teleoperation. Therefore, we propose one more communication channel to transfer the force of the operator to the slave side, then some better results can be obtained in comparison with the early research [11].

The outputs of the Master + operator and the Slave + environment interconnection are defined as follows:

$$\begin{cases} y_m = \bar{P}_1 F_{op} + \bar{P}_2 e_m + \bar{P}_3 \dot{e}_m \\ y_s = \bar{K}_1 F_e + \bar{K}_2 e_s + \bar{K}_3 \dot{e}_s \end{cases} \tag{22}$$

where $e_m = z_m - \hat{z}_s$ and $e_s = z_s - \hat{z}_m$ are the position errors, $\dot{e}_m = \dot{z}_m - \dot{\hat{z}}_s$ and $\dot{e}_s = \dot{z}_s - \dot{\hat{z}}_m$ are the velocity errors in the slave and master sides, and $\bar{K}_1 = \bar{K}_1^T \geq 0,\ \bar{K}_2 = \bar{K}_2^T \geq 0,\ \bar{K}_3 = \bar{K}_3^T \geq 0$ and $\bar{P}_1 = \bar{P}_1^T \geq 0,\ \bar{P}_2 = \bar{P}_2^T \geq 0,\ \bar{P}_3 = \bar{P}_3^T \geq 0$ are the gain matrices.

We define the FR signals of both sides as follows:

$$\begin{cases} \tilde{F}_e(t) = y_s(t) \\ \tilde{F}_{op}(t) = P y_m(t) \end{cases} \tag{23}$$

The signal $\hat{F}_e(t)$ is transmitted to the master side, and $\hat{F}_{op}(t)$ is transmitted to the slave side with communication delays $T_s(t), T_m(t)$.

Concerning (22) and (23), the general formulas of FR signals are given as:

$$\begin{cases} \tilde{F}_e(t) = K_{fb}(\bar{K}_1 F_e + \bar{K}_2 e_s + \bar{K}_3 \dot{e}_s) \\ \tilde{F}_{op}(t) = P_{fw}(\bar{P}_1 F_{op} + \bar{P}_2 e_m + \bar{P}_3 \dot{e}_m) \end{cases} \quad (24)$$

The block of force reflection scheme is shown in Fig.1.

4. DAMPING VALUE MODULATION

One of the control objectives of teleoperation systems is to achieve a good tracking performance of the slave robot during motions in free space, as well as a good contact stability during motions resulting in contact with the environment. The damping of both master and slave may be empirically constructed to provide the desired alterations. As much previous research of force reflecting teleoperation system, when the end-effector of the slave robot is controlled to contact with a hard environment, the tracking performance is not good, especially in low damping cases. Sometimes this makes the system unstable after a short time of contact although it had a good tracking performance in free space before.

The desired damping values of master and slave robots are also the parameters in the control law; they are selected depending on whether the slave is in free space or in contact with an environment. The variable damping values in these cases are assumed to be bounded for the damping modulation that was also introduced in [8].

The master and slave damping matrices are shown as below:

$$B_m = \begin{bmatrix} \tilde{b}_{v_m 1} & 0 \\ 0 & \tilde{b}_{v_m 2} \end{bmatrix}, \; B_s = \begin{bmatrix} \tilde{b}_{v_s 1} & 0 \\ 0 & \tilde{b}_{v_s 2} \end{bmatrix} \quad (25)$$

where $\tilde{b}_{vm1}$, $\tilde{b}_{vm2}$, $\tilde{b}_{vs1}$, $\tilde{b}_{vs2}$ are the variable damping values of master and slave, respectively. These values are modulated according to the distance of slave end-effector from a staring point to the other point on the environment surface (following the y_s-axis). Based on the proposal in [8], if we call $z_{y_{env}}$ to be y-axis position of the environment, then variable damping values are defined following two positions as:

1) $z_{y_{env}} \le 0$:

$$\tilde{b}_{v_i}(z_y) = \begin{cases} \underline{b}_c, & z_y > 0 \\ \Gamma_3 z_y^3 + \Gamma_2 z_y^2 + \Gamma_1 z_y + \Gamma_0, & z_{ymax} \le z_y \le 0 \\ \overline{b}_f, & z_y < z_{ymax} \end{cases} \quad (26)$$

2) $z_{y_{env}} > 0$:

$$\tilde{b}_{v_i}(z_y) = \begin{cases} \underline{b}_c, & z_y < 0 \\ \Gamma_3 z_y^3 + \Gamma_2 z_y^2 + \Gamma_1 z_y + \Gamma_0, & 0 \le z_y \le z_{ymax} \\ \overline{b}_f, & z_y > z_{ymax} \end{cases} \quad (27)$$

here $i = m/s$ and z_y, z_{ymax} are the distances from starting point to the position of the end-effector of the slave and to the contact point in the environment, respectively; $\underline{b}_c$ and $\overline{b}_f$ are lower and upper bound values of damping; The coefficients Γ_i $(i = 1 \div 3)$ are obtained with the constraints of: $\tilde{b}_{v_i}(0) = \underline{b}_c$ and $\tilde{b}_{v_i}(z_{ymax}) = \overline{b}_f$.

5. STABILITY ANALYSIS

This section deals with the stability of the overall teleoperation system that includes master and slave subsystems. We defined a state of this system similar to (21). First, concern the approach in [11], we consider the master subsystem in (1) following the Lemma below.

Lemma 1: The closed-loop master subsystem with state $x_M = (z_m^T, \; \dot{z}_m^T)^T$, input $u_M = (F_{op} - \hat{F}_e)$ and output $y_M = (z_m^T, \; \dot{z}_m^T)^T$ is input-to-state stable (ISS), and also is input-to-output stable (IOS)

Proof: First, consider an ISS-Lyapunov function candidate:

$$V_m = \frac{1}{2}\xi_m^T H_m \xi_m \quad (28)$$

here ξ_m is defined as:

$$\xi_m = (\dot{z}_m - \sigma_2) + \Lambda_m(z_m - \sigma_1)$$

where Λ_m is diagonal matrix gain. We can easily check that $\underline{\alpha}_m(|x_M|) \le V_m \le \bar{\alpha}_m(|x_M|)$ while $\underline{\alpha}_m, \; \bar{\alpha}_m \in \mathcal{K}_\infty$.The time derivative of V_m along trajectories of the system is:

$$\dot{V}_m = \xi_m^T H_m \dot{\xi}_m \quad (29)$$

We consider the FR stabilization algorithm where the velocity measurements are replaced by the estimates obtained using the so-called "dirty derivative" filters, which were also introduced in [11] as follows:

$$\begin{cases} \dot{\sigma}_1 = \sigma_2 - \Lambda_m(z_m - \sigma_1) \\ \dot{\sigma}_2 = H_m^{-1}[-B_m\dot{\sigma}_1 - K_m(z_m - \hat{z}_s)] \end{cases} \quad (30)$$

We have:

$$\dot{\xi}_m = \ddot{z}_m - \dot{\sigma}_2 + \Lambda_m(\dot{z}_m - \dot{\sigma}_1)$$

Substituting $\ddot{z}_m$ from (16) into $\dot{\xi}$ and then replace them in (29) while noticing the formulas of $F_{tel\,m}$ and $F_{ext\,m}$, we get:

$$\begin{aligned} \dot{V}_m =& \xi_m^T H_m \big[H_m^{-1}[-B_m(\dot{z}_m - \sigma_2 + \Lambda_m(z_m - \sigma_1)) \\ &+ F_{ext\,m}] + \Lambda_m \xi_m\big] \\ =& -\xi_m^T(B_m - H_m\Lambda_m)\xi_m + \xi_m^T(F_{op} - \hat{F}_e) \end{aligned} \quad (31)$$

We use Young's quadratic inequality with $|a^T b| \le (\varepsilon/2)|a|^2 + (1/2\varepsilon)|b|^2$ that holds for all $\varepsilon > 0$, therefore we can obtain the following relationship:

$$\xi_m^T(F_{op} - \hat{F}_e) \le \frac{\lambda_{min}(\Lambda_m)}{4}|\xi_m|^2 + \frac{1}{\lambda_{min}(\Lambda_m)}|F_{op} - \hat{F}_e|^2 \quad (32)$$

then we get:

$$\begin{aligned} \dot{V}_m \le& -\xi_m^T(B_m - H_m\Lambda_m)\xi_m + \frac{\lambda_{min}(\Lambda_m)}{4}|\xi_m|^2 \\ &+ \frac{1}{\lambda_{min}(\Lambda_m)}|F_{op} - \hat{F}_e|^2 \end{aligned} \quad (33)$$

Now, let $\gamma_\Lambda \in \mathcal{K}$ be defined as: $\gamma_\Lambda = \lambda_{min}(\Lambda_m)/4$ then we can choose $\Lambda_m = \Lambda_m^T > 0$ with bounded $\Lambda_m < B_m/H_m$ to satisfy $\gamma_\Lambda(\Lambda_m) \le 4[B_m - H_m\Lambda_m]$.

Applying the results of Sontag and Wang (1995) [15] (see Appendix) and in [14], the subsystem is input-to-state stable with the state $(z_m^T, \; \dot{z}_m^T)^T$. Here the output is similar to the input $(y = x)$, so the subsystem is also input-to-output stable. ■

Now, we consider the slave-environment interconnection with the slave subsystem.

Lemma 2: State of the closed-loop slave subsystem is assumed as: $x_S = (\tilde{z}_s^T, \xi_s^T, x_e^T)^T$, and input: $u_S = ((z_1^*)^T, (z_2^*)^T, \varsigma_1^T, \varsigma_2^T, \tilde{z}_s^T, \hat{F}_{op}^T)^T$. We suppose the environment dynamics (3) satisfy Assumption 1. Then there exits

$\gamma_\Lambda \in \mathscr{K}$ such that if $\lambda_{min}(K_s) \geq \gamma_\Lambda(\|\Lambda_s - \Lambda_{env}\|)$, then the slave-environment interconnection is input-to-state stable.

Proof: First, consider the ISS-Lyapunov function candidate:

$$V_s = \frac{1}{2}\xi_s^T H_s \xi_s + \frac{1}{2}k_z \tilde{z}_s^T \tilde{z}_s + V_e \tag{34}$$

where V_e is introduced in Assumption 1, $k_z > 0$ is a constant to be determined, $\tilde{z}_s = z_s - \varsigma_1$ is the slave position error estimation. And one can easily check that V_s satisfies the inequality $\underline{\alpha}_s(|x_S|) \leq V_s \leq \bar{\alpha}_s(|x_S|)$ while $\underline{\alpha}_s,\ \bar{\alpha}_s \in \mathscr{K}_\infty$. Calculating the time derivative of V_s along the trajectories of the system as:

$$\dot{V}_s = \xi_s^T H_s \dot{\xi}_s + k_z \tilde{z}_s^T \dot{\tilde{z}}_s + \dot{V}_e \tag{35}$$

Similar to the master subsystem, we set:

$$\xi_s = (\dot{z}_s - \varsigma_2) + \Lambda_s(z_s - \varsigma_1);$$
$$\dot{\xi}_s = (\ddot{z}_s - \dot{\varsigma}_2) + \Lambda_s(\dot{z}_s - \dot{\varsigma}_1)$$

and using the "dirty-derivative" filter as follows:

$$\begin{cases} \dot{\varsigma}_1 = \varsigma_2 - \Lambda_s(z_s - \varsigma_1) \\ \dot{\varsigma}_2 = H_s^{-1}[-B_s\dot{\varsigma}_1 - K_s(z_s - \hat{z}_m)] \end{cases} \tag{36}$$

where Λ_s is diagonal matrix gain. We consider (16) to get $\ddot{z}_s$ and substitute (36) into $\dot{\xi}_s$ while noticing the formula of $F_{tel\,s}$ in (18), we receive:

$$\dot{\xi}_s = H_s^{-1}[-B_s(\dot{z}_s - \dot{\varsigma}_1) + F_{ext\,s}] + \Lambda_s \xi_s \tag{37}$$

Considering the fact that:

$$\dot{\tilde{z}}_s = -\Lambda_s \tilde{z}_s + \xi_s + \varsigma_2 - \dot{\varsigma}_1 \tag{38}$$

and substituting (37), (38) into (35) and concerning $\dot{V}_e$ in Assumption 1, the formula of $F_{ext\,s}$ in (17), we get:

$$\begin{aligned} \dot{V}_s \leq & -\xi_s^T(B_s - H_s\Lambda_s)\xi_s - k_z\tilde{z}_s^T\Lambda_s\tilde{z}_s - \alpha_{3e}|x_e|^2 \\ & + \hat{F}_{op}^T\xi_s + F_e^T(s_e - \xi_s) + k_z\tilde{z}_s^T\xi_s + k_z\tilde{z}_s^T(\varsigma_2 - \dot{\varsigma}_1) \end{aligned} \tag{39}$$

Modifying the formula of ξ_s as follows:

$$\xi_s = (\dot{z}_s - \varsigma_2) + \Lambda_{env}(z_s - \varsigma_1) - (\Lambda_{env} - \Lambda_s)(z_s - \varsigma_1)$$

Considering the equation (5), we receive:

$$(s_e - \xi_s) = \underbrace{\varsigma_2 - z_2^* + \Lambda_{env}(\varsigma_1 - z_1^*)}_{\Omega_1} + \underbrace{(\Lambda_{env} - \Lambda_s)}_{\Omega_2}\tilde{z}_s \tag{40}$$

Using the formulas: $z_s = \tilde{z}_s + \varsigma_1,\ \dot{z}_s = \xi_s - \Lambda_s\tilde{z}_s + \varsigma_2$; and noticing the inequality (4), we get:

$$|F_e| \leq a(|x_e| + \| \Lambda_s + I \| \, |\tilde{z}_s| + |\xi_s| + |\varsigma_1| + |\varsigma_2|) + b \tag{41}$$

Combining (40) and (41), we get the estimate:

$$\begin{aligned} |(s_e - \xi_s)^T F_e| \leq & \; a|x_e||\Omega_1| + a \| \Lambda_s + I \| \, |\tilde{z}_s||\Omega_1| \\ & + a|\xi_s||\Omega_1| + a(|\varsigma_1| + |\varsigma_2|)|\Omega_1| + b|\Omega_1| \\ & + a|x_e| \, \| \Omega_2 \| \, |\tilde{z}_s| + a \| \Lambda_s + I \| \| \Omega_2 \| \, |\tilde{z}_s|^2 \\ & + a \| \Omega_2 \| \, |\xi_s||\tilde{z}_s| + a \| \Omega_2 \| \, (|\varsigma_1| + |\varsigma_2|)|\tilde{z}_s| \\ & + b \| \Omega_2 \| \, |\tilde{z}_s| \end{aligned} \tag{42}$$

Using the fact that $(\varsigma_2 - \dot{\varsigma}_1) = \Lambda_s(z_s - \varsigma_1)$, and applying Young's quadratic inequality form, we can obtain the following set of bounds:

$$\hat{F}_{op}^T\xi_s \leq \frac{\lambda_{min}(\Lambda_s)}{4}|\hat{F}_{op}|^2 + \frac{1}{\lambda_{min}(\Lambda_s)}|\xi_s|^2 \tag{43}$$

$$k_z\tilde{z}_s^T\xi_s \leq k_z\left(\frac{\lambda_{min}(\Lambda_s)}{4}|\tilde{z}_s|^2 + \frac{1}{\lambda_{min}(\Lambda_s)}|\xi_s|^2\right) \tag{44}$$

$$\begin{aligned} & k_z\tilde{z}_s^T\Lambda_s(z_s - \varsigma_1) \leq \\ & \qquad \frac{k_z\lambda_{min}(\Lambda_s)}{4}|\tilde{z}_s|^2 + \frac{k_z\Lambda_s^2}{\lambda_{min}(\Lambda_s)}|z_s - \varsigma_1|^2 \end{aligned} \tag{45}$$

$$a|x_e||\Omega_1| \leq \frac{\alpha_{3e}}{4}|x_e|^2 + \frac{a^2}{\alpha_{3e}}|\Omega_1|^2 \tag{46}$$

$$\begin{aligned} & a \| \Lambda_s + I \| \, |\tilde{z}_s||\Omega_1| \leq \\ & \qquad \frac{k_z\lambda_{min}(\Lambda_s)}{4}|\tilde{z}_s|^2 + \frac{a^2\|\Lambda_s + I\|^2}{k_z\lambda_{min}(\Lambda_s)}|\Omega_1|^2 \end{aligned} \tag{47}$$

$$a|\xi_s||\Omega_1| \leq \frac{\lambda_{min}(K_s)}{4}|\xi_s|^2 + \frac{a^2}{\lambda_{min}(K_s)}|\Omega_1|^2 \tag{48}$$

$$a|x_e| \, \| \Omega_2 \| \, |\tilde{z}_s| \leq \frac{\alpha_{3e}}{4}|x_e|^2 + \frac{a^2}{\alpha_{3e}} \| \Omega_2 \|^2 \, |\tilde{z}_s|^2 \tag{49}$$

$$a \| \Omega_2 \| \, |\xi_s||\tilde{z}_s| \leq \| \Omega_2 \| \, (a^2|\xi_s|^2 + \tfrac{1}{4}|\tilde{z}_s|^2) \tag{50}$$

$$\begin{aligned} & a \| \Omega_2 \| \, (|\varsigma_1| + |\varsigma_2|)|\tilde{z}_s| \leq \\ & \qquad \| \Omega_2 \| \, (a^2(|\varsigma_1| + |\varsigma_2|)^2 + \tfrac{1}{4}|\tilde{z}_s|^2) \end{aligned} \tag{51}$$

$$b \| \Omega_2 \| \, |\tilde{z}_s| \leq \| \Omega_2 \| \, (b^2 + \tfrac{1}{4}|\tilde{z}_s|^2) \tag{52}$$

Combining (43)-(52) and (39), (42), we get:

$$\begin{aligned} \dot{V}_s \leq & -\xi_s^T(B_s - H_s\Lambda_s)\xi_s - k_z\tilde{z}_s^T\Lambda_s\tilde{z}_s - \frac{\alpha_{3e}}{2}|x_e|^2 \\ & + \Big[\frac{1 + k_z}{\lambda_{min}(\Lambda_s)} + a^2 \| \Omega_2 \| \Big]|\xi_s|^2 \\ & + \Big[\Big(a \| \Lambda_s \| + a + \frac{3}{4}\Big) \| \Omega_2 \| + \frac{a^2}{\alpha_{3e}} \| \Omega_2 \|^2 \Big]|\tilde{z}_s|^2 \\ & + \Big[\frac{a^2}{\alpha_{3e}} + \frac{a^2 \| \Lambda_s + I \|^2}{k_z\lambda_{min}(\Lambda_s)} + \frac{a^2}{\lambda_{min}(K_s)}\Big]|\Omega_1|^2 \\ & + \big[a(|\varsigma_1| + |\varsigma_2|) + b\big]|\Omega_1| + \| \Omega_2 \| \, \big[(a^2(|\varsigma_1| + |\varsigma_2|)^2 \\ & + b^2\big] + \frac{\lambda_{min}(\Lambda_s)}{4}|\hat{F}_{op}|^2 + \frac{k_z\Lambda_s^2}{\lambda_{min}(\Lambda_s)}|z_s - \varsigma_1|^2 \end{aligned} \tag{53}$$

Now, let $\gamma_{1\Lambda},\ \gamma_{2\Lambda} \in \mathscr{K}_\infty$ be defined for each $s \geq 0$ as follows:

$$\gamma_{1\Lambda}(s) = \frac{\big(a \| \Lambda_s \| + a + (3/4)\big)s + \big(a^2/\alpha_{3e}\big)s^2}{2\lambda_{min}(\Lambda_s)}$$

$$\gamma_{2\Lambda}(s) = \frac{s}{\lambda_{min}(\Lambda_s)} + a^2\gamma_{1\Lambda}^{-1}(s)$$

and consider the small gain $\gamma_\Lambda = \gamma_{1\Lambda}(s) \circ \gamma_{2\Lambda}(s) \in \mathscr{K}_\infty$. Similar to the Lemma 1, we can choose $\Lambda_s = \Lambda_s^T > 0$ with bounded $\Lambda_s < B_s/H_s$, and $K_s = K_s^T > 0$, $k_z > 0$ satisfying:

$$\lambda_{min}(K_s) \geq \gamma_{2\Lambda}(k_z) \geq \gamma_\Lambda(\| \Lambda_{env} - \Lambda_s \|) \tag{54}$$

implies that:

$$\begin{aligned} \dot{V}_s \leq & -\xi_s^T(B_s - H_s\Lambda_s)\xi_s - k_z\tilde{z}_s^T\Lambda_s\tilde{z}_s - \frac{\alpha_{3e}}{2}|x_e|^2 \\ & + \Big[\frac{a^2}{\alpha_{3e}} + \frac{a^2 \| \Lambda_s + I \|^2}{k_z\lambda_{min}(\Lambda_s)} + \frac{a^2}{\lambda_{min}(K_s)}\Big]|\Omega_1|^2 \\ & + \big[a(|\varsigma_1| + |\varsigma_2|) + b\big]|\Omega_1| + \| \Omega_2 \| \, \big[(a^2(|\varsigma_1| + |\varsigma_2|)^2 \\ & + b^2\big] + \frac{\lambda_{min}(\Lambda_s)}{4}|\hat{F}_{op}|^2 + \frac{k_z\Lambda_s^2}{\lambda_{min}(\Lambda_s)}|z_s - \varsigma_1|^2 \end{aligned} \tag{55}$$

Similar to the master subsystem, the results in [15] of Sontag and Wang are used (see Appendix). Then the slave subsystem is input-to-state stable with state $(\tilde{z}_s^T,\ \xi_s^T,\ x_e^T)^T$. ■

Based on the Lemma 1 and Lemma 2, the following theorem concerning stability properties of the closed-loop system is obtained.

Theorem 1: Consider the force-reflecting teleoperator system (1) and (16) with FR algorithm (24). Suppose the environment dynamics satisfy Assumption 1, and the communication delays $T_{m/s}(\cdot)$ satisfy Assumption 3, there exists $\gamma_\Lambda(\cdot) \in \mathscr{K}$ such that $\lambda_{min}(K_s) \geq \gamma_\Lambda(\| \Lambda_s - \Lambda_{env} \|)$ implies that: for the FR algorithm (24), the overall teleoperation system is input-to-state stable (ISS).

Proof: Now we can combine the above presented results and the consecutive application of the IOS small gain theorem in [11]. Indeed, denote by $\gamma_{[u_M \to y_M]}(\cdot) \in \mathscr{K}$ the ISS gain of the closed-loop master subsystem whose existence is guaranteed by Lemma 1. And also, we let $\gamma_{[u_S \to x_S]}(\cdot) \in \mathscr{K}$ be the IOS gain of the closed-loop slave + environment subsystem (3). Choose $\alpha^*(\cdot) \in \mathscr{K}_\infty$ such that the inequality:

$$\alpha^* \circ \gamma_{[u_S \to x_S]}(\cdot) \circ \gamma_{[u_M \to y_M]}(\cdot)(s) < s \tag{56}$$

holds for all $s > 0$. Applying the IOS small gain theorem, the overall teleoperation system is input-to-state stable for any $\alpha \in \mathscr{N}$ satisfying $\alpha(s) \leq \alpha^*$ for all $s \geq 0$. ■

6. EVALUATION BY CONTROL EXPERIMENTS

In this section, we verify the efficacy of the proposed FR teleoperation. The experiments were carried out on a pair of identical direct-drive planar robots with 2 links revolute-joints. The inertia matrices and the Coriolis matrices are identified as:

$$M_i = \begin{bmatrix} M_{i1} + 2R_i cos(q_{i2}) & M_{i2} + R_i cos(q_{i2}) \\ M_{i2} + R_i cos(q_{i2}) & M_{i2} \end{bmatrix},$$

$$C_i = \begin{bmatrix} -R_i sin(q_{i2})\dot{q}_{i2} & -R_i sin(q_{i2})(\dot{q}_{i1} + \dot{q}_{i2}) \\ R_i sin(q_{i2})\dot{q}_{i1} & 0 \end{bmatrix}$$

where $M_{i1} = 0.366\ kgm^2$, $M_{i2} = 0.0291\ kgm^2$, $R_i = 0.0227\ kgm^2$; $l_{i1} = l_{i2} = 0.2\ m$, with $i = m,\ s$. The remote environment on the slave side is a hard aluminum wall and its surface is covered by soft rubber as shown in Fig. 3. The contact forces between the end-effector of the slave robot with the environment are shown in Fig. 4. We also receive joint angle values from encoders in each joint of the robots, and measure the operational and environment reflecting forces by using the force sensors at the end-effectors of the robots (F_{Sx}, F_{Sy}). For implementation of the controllers and communication lines, we utilise a dSPACE digital control system (dSPACE Inc.). All experiments have been done with the artificial time varying communication delays as:

$$T_m(t) = 0.2 sin 0.3t + 0.3\ [s]$$

$$T_s(t) = 0.2 sin 0.3t + 0.3\ [s]$$

We can see the above communication delays also satisfy Assumption 3 with $T_{m/s}(\cdot) : R \to R^+$. The experiment setup is shown in Fig. 3, here the slave is controlled to contact the surface of environment in $(x_1,\ y_1)$

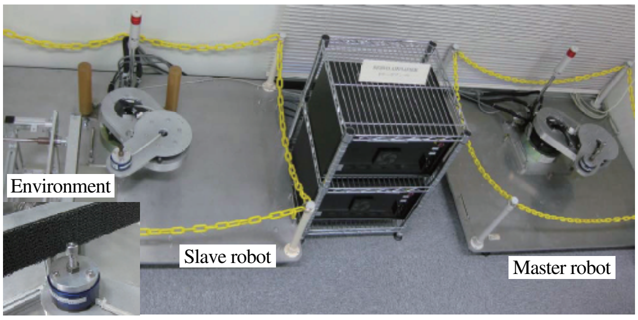

Fig. 3 Experimental setup.

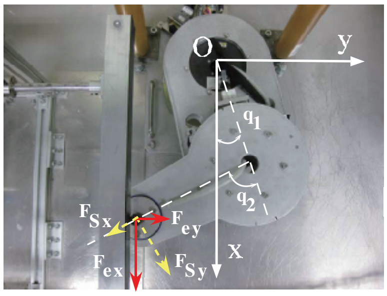

Fig. 4 Force in the contact task.

from initial position $(x_0,\ y_0)$. The initial joint angles of the robots are chosen to satisfy Assumption 1, then we set $q_1 = 45^0$, $q_2 = -90^0$ and they are equivalent in task space with $x_0 = 0.2828\ m$, $y_0 = 0.0\ m$. The contact position is set as: $x_1 = 0.2828\ m$, $y_1 = -0.16\ m$. The controller parameters are selected as follows to guarantee the ISS conditions in Lemma 1 and Lemma 2:

$$H_m = H_s = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix},\ B_m = B_s = \begin{bmatrix} \tilde{b}_{v_{m/s}1} & 0 \\ 0 & \tilde{b}_{v_{m/s}2} \end{bmatrix};$$

$$K_m = \begin{bmatrix} 45 & 0 \\ 0 & 45 \end{bmatrix},\ K_s = \begin{bmatrix} 95 & 0 \\ 0 & 95 \end{bmatrix};$$

$$\Lambda_m = \begin{bmatrix} 0.015 & 0 \\ 0 & 0.0015 \end{bmatrix},\ \Lambda_s = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.8 \end{bmatrix}$$

In this control task, the varying damping values are received from (26) with following parameters:

$z_{ymax} = 0.12,\ \bar{b}_f = 45,\ \underline{b}_c = 10$

and some gains of the force reflection scheme are chosen: $\bar{P}_1 = \bar{K}_1 = 0.5$, $\bar{P}_2 = \bar{K}_2 = 0.03$, $\bar{K}_3 = 0.002$, $\bar{K}_3 = 0.02$, $K_{fw} = 0.5$, $K_{fb} = 1.5$.

Two kinds of experimental conditions are given as:

Case 1. The slave moves without any contact.

Case 2. The slave moves in contact with the environment.

The Figs. 5-10 show the results with two cases of experimental conditions. Figs. 5-7 show the results of Case 1. We can see from Fig. 5, the free movement of slave robot is achieved accurately the movement of the master robot. In this case there is only the force exerted on the master by the human operator. Since the end-effector of the slave robot does not contact with the environment, obviously there is not the reflecting force from there.

Figs. 8-10 show the results of the second case. From

Fig. 8 we can see that after moving in free space (0-18[sec]), the slave robot contacts with the environment (18-35[sec]), the reflecting force appears and increases while the human pushes an increasing force on the master robot. As shown in Fig. 9, this contact force is faithfully reflected to the master side. The human operator can perceive the environment through the reflection force, however in this case the position error is larger than the error in free movement case of slave robot. When the slave robot departs from the environment and the human does not exert more force on the master (35-50[sec]), the position error becomes smaller.

In Fig. 7 and Fig. 10, the varying damping values of the master and the slave robots are shown in 2 cases; the environment position is set with $z_{y_{env}} \leq 0$; these values depend on the distance (following the y_s-axis) from the starting point (x_0, y_0) to the current position of the end-effector of the slave robot. We can set the upper and lower bound values of damping, they depend on the distance from the starting point to the surface of the environment. The overall system guarantees the ISS and achieves contact stability and also good transparency while the damping values satisfy the conditions from (33). In the experimental task, when the end-effector of the slave robot contacts with the surface of the environment, the damping achieves the upper bound value (see Fig. 10) to keep the contact stability.

7. CONCLUSION

In this paper, we proposed an impedance control input based on a new force reflection (FR) algorithm for bilateral teleoperation with time varying delays. In this proposed strategies, besides using the new FR scheme we used varying damping to improve contact stability and transparency of teleoperation with the effective tracking performance in comparison with the previous research. To analyze stability, the input-to-output stability (IOS) small gain theorem was used to show the overall FR teleoperation system to be input-to-state stable (ISS). Finally, several experimental results showed the effectiveness of our proposed methods.

Our future work entails that the control method will be improved more in harder environment contact without destabilization and deterioration of the transparency.

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APPENDIX

Consider the following general system:

$$\dot{x} = f(x,\ y) \tag{57}$$

here $f \in R^n \times R^m \rightarrow R^n$ is continuously differentiable and satisfies $f(0,0) = 0$.

Definition 1: The system (57) is (globally) input-to-state stable (ISS) if there exist a $\mathscr{KL}$-function $\beta : R_{\geq 0} \times$

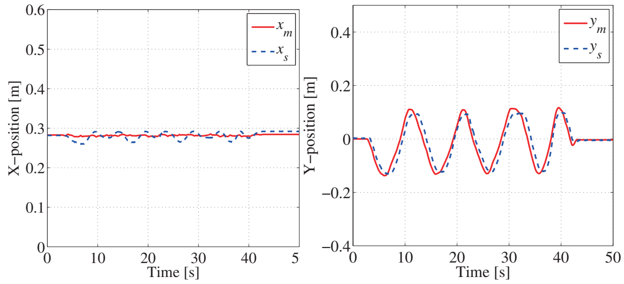

Fig. 5 Position data in free space (Case 1).

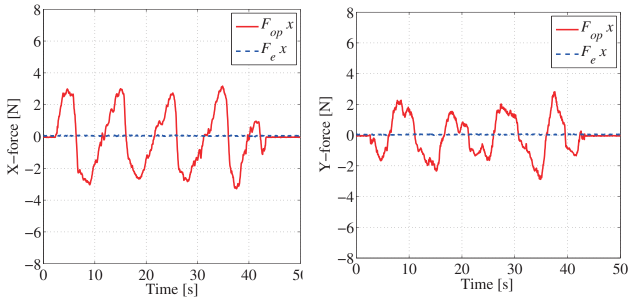

Fig. 6 Force data in free space (Case 1).

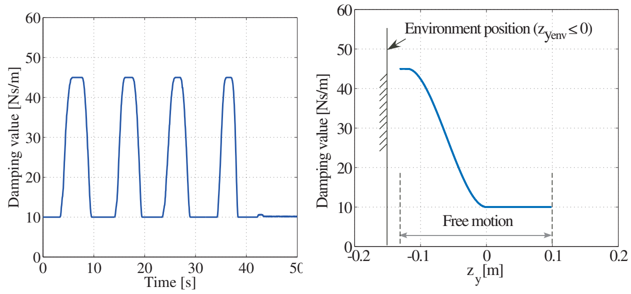

Fig. 7 Varying damping values in free space (Case 1).

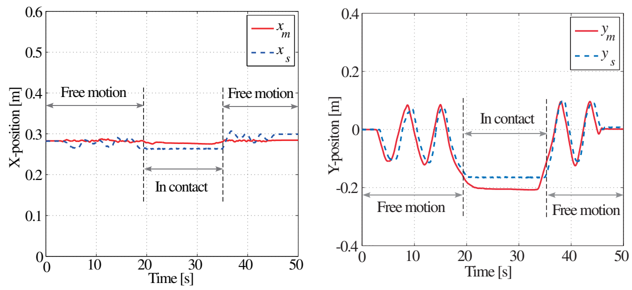

Fig. 8 Position data in contact with environment(Case 2).

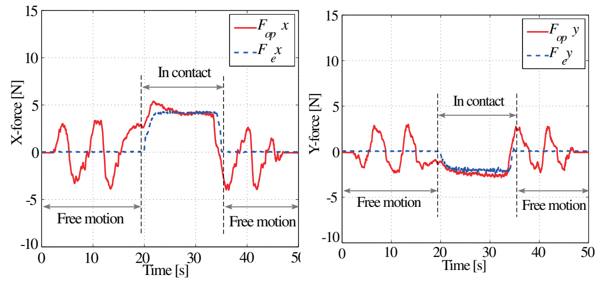

Fig. 9 Force data in contact with environment (Case 2).

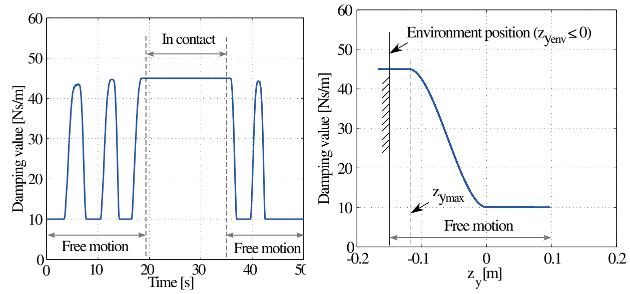

Fig. 10 Varying damping values in contact with environment (Case 2).

$R_{\geq 0} \to R$ and a $\mathcal{K}$-function γ such that for each input $u \in L_\infty^m$ and each $\xi \in R^n$, it holds that:

$$|x(t,\xi,u)| \leq \beta(|\xi|,t) + \gamma(\|u\|) \tag{58}$$

for each $t \geq 0$.

Definition 2: A smooth function $V : R^n \to R_{\geq 0}$ is called ISS-Lyapunov function for system (57) if there exits $\mathcal{K}_\infty$-function α_1, α_2 and $\mathcal{K}$-function α_3 and χ, such that:

$$\alpha_1(|\xi|) \leq V(\xi) \leq \alpha_2(|\xi|) \tag{59}$$

for any $\xi \in R^n$ and

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(t,\xi,u) \leq -\alpha_3(|\xi|) \tag{60}$$

for any $\xi \in R^n$ and any $u \in R^m$ so that $|\xi| \geq \chi(|u|)$.

Remark 1: A smooth function V is an ISS-Lyapunov function for (57) if and only if there exits $\alpha_i \in \mathcal{K}_\infty$ $(1 \leq i \leq 4)$ such that (57) holds, and

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(t,\xi,u) \leq -\alpha_3(|\xi|) + \alpha_4(|u|) \tag{61}$$

This provides a "dissipation" type of characterization for the ISS property. Clearly (61) implies (60). Assume now that (60) holds with some $\alpha_3 \in \mathcal{K}_\infty$ and $\chi \in \mathcal{K}$. Let $\alpha_4(r) = max\{0, \hat{\alpha}_4(r)\}$ where $\hat{\alpha}_4(r) = max\{(\partial V/\partial t + \partial V/\partial x) f(t,\xi,u) + \alpha_3(\chi(|u|)) : |u| \leq r, \xi \leq \chi(r)\}$. Then α_4 is continuous and $\alpha_4(0) = 0$, and one can assume that α_4 is a $\mathcal{K}$-function (majorize it by one if it is not). Note then that (61) is holds because $\alpha_4(r) \geq sup_{|u|=r}(\partial V/\partial t + \partial V/\partial x) f(t,\xi,u) + \alpha_3(|\xi|)$ (consider the two separate cases $|\xi| \geq \chi(|u|)$ and $|\xi| \leq \chi(|u|)$).