

Active Vibration Control for Flexible Structures Using a Wave-Absorbing Control Method*

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This paper proposes active vibration control of flexible structures using wave-absorbing method. The method is based on the active restraint of the formation of standing waves which are created in continuous vibration. The Euler-Bernoulli beam is used as the control object. Considering the general solution of the wave equation, we obtain an active control method which eliminates the reflection waves at the boundary. The effectiveness of the restraint of the resonance is shown in frequency domains. We then apply the method to impulse responses which have wide frequency domains. Consequently, we show that the vibration of the beam is effectively and quickly controlled before the standing waves are formed.

Key Words: Active Wave Control, Flexible Structure, Vibration Control

1. Introduction

There are many reports⁽¹⁾ on vibration control methods for flexible structures. The mathematical models of flexible structures are expressed, in general, by partial differential equations. Some cases⁽²⁾ are dealt with as distributed parameter systems, but often the modal expansion method is used, which approximates the distributed parameter system by a system of finite dimensions. Recently, however, the limitations of the modal expansion method in controlling high dimension modes are being reached due to the requirements of large structures and high-precision

control devices.

Therefore, we propose a new wave-absorbing control method for a wide frequency band. This method regards the vibration as superposed propagation waves, changes the reflection characteristics of the travelling waves at the control position and tries to absorb their energy. It designs the compensator without the spillover which is ordinarily generated by the modal expansion method, because the partial differential equation of the mathematical model is not approximated by a system of finite dimensions. This concept was first applied to a control problem by von Flotow and Schafer⁽³⁾⁻⁽⁵⁾. Their method was to change the reflection coefficient matrix between the incoming wave mode and the outgoing wave mode to a suitable form. After that, they introduced the H_2 -optimal control of power flow from the system⁽⁶⁾. An H^∞ -optimal control of power flow is then discussed by MacMartin and Hall⁽⁷⁾, for design of a compensator for general structures. Von Flotow's method is expanded by Fujii and co-workers^{(8),(9)} to the non-colocate case. They proposed a method for obtaining the H^∞ norm of the reflection coefficient matrix, and applied it to the problem of the impulse response⁽¹⁰⁾.

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Tanaka and co-workers⁽¹¹⁾⁻⁽¹⁴⁾ examined the formation process of vibration modes and proposed the active sink method which inactivated every vibration mode of the structures.

Considering that the standing waves generated in the structure play an important role in continuous vibration, we control the formation of the standing waves actively using the wave-absorbing method. First, we obtain the sine-wave general solution for the governing equation of the structure. Then we derive the forward and backward wave solutions for formation of the standing waves, and construct a controller which absorbs the incoming waves and eliminates the outgoing waves at the control position.

This paper uses the Euler-Bernoulli beam as the control object which generates elastic bending waves with a dispersive property. The wave-absorbing control is first applied to the restraint of resonance in frequency domains. Then, in the case of concentrated impulse load which has many vibration modes, the restraint of the vibration is confirmed before the standing waves are formed in the structures. Finally, as an application of the result, the control method for arbitrary distributed impulse load is proposed.

2. The Bending Vibration of the Euler-Bernoulli Beam

A thin rod or a belt-shaped structure undergoing bending action is generally called a beam. The vibration of the traverse motion to the neutral axis of the beam is called "the bending vibration of the beam". Its basic governing equations are, based on different assumptions, classified into several types such as Euler-Bernoulli, Rayleigh, Timoshenko and so on. This paper deals with the most simple model called the Euler-Bernoulli beam.

The basic assumptions of the Euler-Bernoulli beam are that the material of the beam is homogeneous; the deflections are perfectly elastic and small; and the stress is proportional to the strain; the local rotational moment and the inertial force from the shear deformation are neglected. The governing equation is then given by

$$EI \frac{\partial^4 w(x, t)}{\partial x^4} + \rho A \frac{\partial^2 w(x, t)}{\partial t^2} = q \quad (1)$$

where x denotes the coordinate along the neutral axis of the beam, t the time, $w(x, t)$ the lateral deflection (cf. Fig. 1), EI the bending rigidity, ρA the linear density and the distributed force. Here we assume that the length, the width, the thickness, the bending rigidity, and q the linear density of the beam are 3.0 m, 0.1 m, 1.0 mm, 1.3 Nm², and 0.8 kg/m, respectively.

If the distributed force is zero, Eq. (1) is homogeneous; then, it has the following general sine-wave

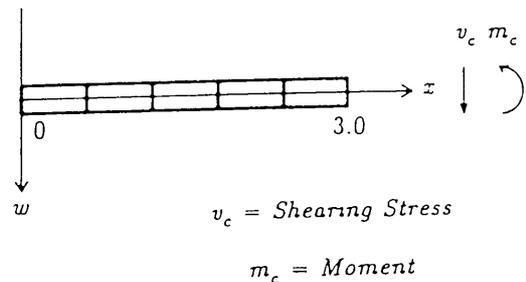


Fig. 1 Beam model

solution of the angular frequency, ω

$$w(x, t) = \bar{a}_1 e^{ikhx} e^{i\omega t} + \bar{a}_2 e^{kx} e^{i\omega t} + \bar{\beta}_1 e^{-ikhx} e^{i\omega t} + \bar{\beta}_2 e^{-kx} e^{i\omega t}, \quad (2)$$

where the relation between angular frequency and wave number k is $\omega = ak^2$ ($a = \sqrt{EI/\rho A}$). By setting

$$\begin{aligned} a_1 &= \bar{a}_1 e^{ikhx} e^{i\omega t}, \\ a_2 &= \bar{a}_2 e^{kx} e^{i\omega t}, \\ \beta_1 &= \bar{\beta}_1 e^{-ikhx} e^{i\omega t}, \end{aligned}$$

and

$$\beta_2 = \bar{\beta}_2 e^{-kx} e^{i\omega t}, \quad (3)$$

we find that α_1 , β_1 , β_2 , and α_2 represent the bending wave travelling in the negative direction of the x axis, the one in the positive direction of the x axis, the near field at $x=0$, and the near field at $x=1$, respectively. (Some papers regard α_1 and α_2 as the waves in the $-x$ direction, and β_1 and β_2 the waves in the $+x$ direction.)

Here, the phase velocity is $c_p = \sqrt{a\omega}$, that is, it has a dispersive property, and the higher phase velocity is associated with the higher angular frequency.

3. Formulation of the Wave-Absorbing Control Method

Bending moment m_c and shear force v_c are used as the boundary control variables. By assuming that the bending moment and the shear force are the feedback of the velocity component of the slope θ ($= \partial w / \partial x$) and the deflection w , respectively, we obtain the following:

$$m_c = EI\mu \frac{\partial \theta}{\partial t}, \quad (4)$$

$$v_c = -EI\nu \frac{\partial w}{\partial t}, \quad (5)$$

where μ and ν are the coefficients of the feedback. (These are convenient forms for discussing Lyapunov stability; see Appendix 1.) Here, we have

$$m_c = -EI \frac{\partial^2 w}{\partial x^2}, \quad (6)$$

$$v_c = \frac{\partial m_c}{\partial x}, \quad (7)$$

so Eq. (6) and Eq. (7) give

$$\frac{\partial^2 w}{\partial x^2} + \mu \frac{\partial^2 w}{\partial t \partial x} = 0, \quad (8)$$

$$\frac{\partial^3 w}{\partial x^3} - \nu \frac{\partial w}{\partial t} = 0. \quad (9)$$

Substituting these equations into Eq.(2) gives

$$(-k^2 - k\omega\mu)\alpha_1 + (k^2 + ik\omega\mu)\alpha_2 + (-k^2 + k\omega\mu)\beta_1 + (k^2 - ik\omega\mu)\beta_2 = 0, \quad (10)$$

$$(-ik^3 - i\omega\nu)\alpha_1 + (k^3 + i\omega\nu)\alpha_2 + (ik^3 + i\omega\nu)\beta_1 + (-k^3 - i\omega\nu)\beta_2 = 0, \quad (11)$$

In this paper the control position is placed at the right end of the beam. At the right end the term of β_2 is much smaller than the other terms because of $e^{-kt} \approx 0$. If $\mu = k/\omega$ and $\nu = k^3/\omega$, the coefficients of β_1 vanish in both of the above equations. Then, we get $\alpha_1 = \alpha_2 = 0$. That is, the outgoing waves (which travel in the negative direction) are eliminated. Therefore, Eq. (4) and Eq.(5) reduce to

$$m_c = EIa^{-1/2}\omega^{-1/2} \frac{\partial \theta}{\partial t}, \quad (12)$$

$$v_c = -EIa^{-3/2}\omega^{1/2} \frac{\partial w}{\partial t}. \quad (13)$$

These results give the right end conditions which will eliminate the outgoing waves for particular angular velocity. By use of control variables m_c and v_c , the formation of the standing wave of angular velocity ω may be restrained.

In the case of the left end control, only the signs of μ , ν have to be altered.

The moving point compliance under controlled conditions is compared with the one under uncontrolled conditions. Figure 2 shows the moving point compliance under controlled and uncontrolled conditions at the right end when the left end is moved. The dotted and solid lines represent the results under uncontrolled and the controlled conditions, respectively.

Figure 2 shows restraint of resonance in all frequency domains. Thus, this control method can restrain the formation of standing waves. Note that it deals, however, with particular angular velocity vibration. In the next section, we examine the problem of

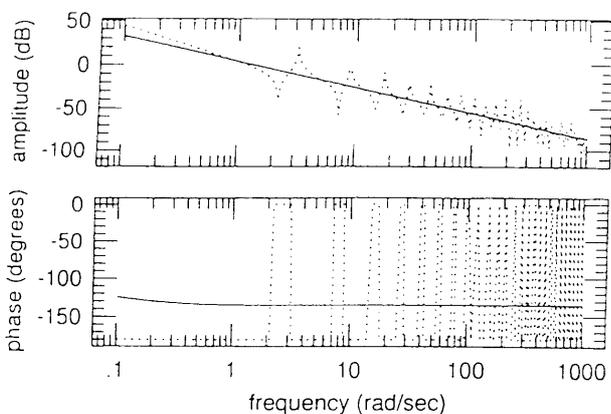


Fig. 2 Moving point compliance

wide frequency vibration mode.

4. Impulse Response

4.1 Concentrated impulse response

Here we consider the case of a beam which has a clamped left end and a free right end, which is struck by a concentrated impulse load (cf. Fig. 3). The control position is placed at the right end, as before.

The control methods of Eqs. (12) and (13) cannot simply be used to control vibrations with several modes, because they are dispersive and their feedback coefficients are functions of angular velocity ω . It is impossible to treat in the frequency domains by applying the Laplace transform because the transmission function in this case is irregular.

We thus examine the propagation of the bending wave in the Euler-Bernoulli beam. In this beam a high-frequency bending wave travels fast. When the beam is struck by an impulse load, bending waves, which are stress waves, propagate throughout the beam, and the form of the waves is then gradually distorted due to their dispersive characteristics. We regard these bending waves as being superposed by several angular frequency waves. When an impulse load is applied, the wave with infinite frequency theoretically reaches the control position first, followed one after another by the waves with lower frequencies. For example, the wave of angular velocity

$$\omega = \frac{l_0^2}{at^2} \quad (14)$$

should reach the position l_0 which is distant from the impulse load point at time t after the initial impulse time.

Consider the kinematic model in Fig. 3. Among the waves which reach the control position from the impulse load point, there exist two kinds of waves, those coming directly from the impulse point and those reflected from the lefthand clamped boundary. Therefore, the relations among the Eqs.(12), (13) and (14) give the following wave-absorbing control method for the direct waves from the impulse point :

$$m_c = EI \frac{t}{l-l_1} \frac{\partial \theta}{\partial t} \quad (15)$$

and

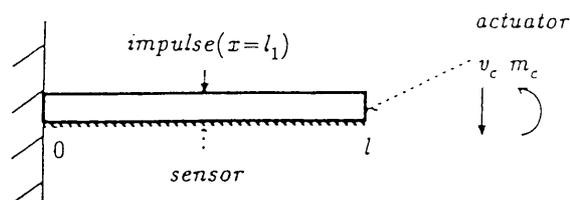


Fig. 3 Kinematic model for impulse response

$$v_c = -EI \frac{l-l_1}{a^2 t} \frac{\partial w}{\partial t}; \tag{16}$$

and for the reflection waves from the left side :

$$m_c = EI \frac{t}{l+l_1} \frac{\partial \theta}{\partial t}; \tag{17}$$

and

$$v_c = -EI \frac{l+l_1}{a^2 t} \frac{\partial w}{\partial t}, \tag{18}$$

respectively. Then the control method for both kinds of waves is assumed as

$$m_c = c_m(t) \frac{\partial \theta}{\partial t} \tag{19.1}$$

where

$$c_m(t) = EI \frac{lt}{(l-l_1)(l+l_1)}, \tag{19.2}$$

and

$$v_c = -c_v(t) \frac{\partial w}{\partial t} \tag{20.1}$$

where

$$c_v(t) = EI \frac{l}{a^2 t}. \tag{20.2}$$

The above equations are stable according to Lyapunov's direct method (see Appendix 1), because both $c_m(t)$ and $c_v(t)$ are positive.

Here we apply the above method to the case that the impulse is loaded at the center of the beam and analyze numerically using the central difference method on the time and the space. Because, from Eq. (20.2), immense control force (shear force) is needed when the time t is very small, we do not employ Eqs. (19.2) and (20.2) from the initial time, but we fix the time t_0 as 1.18×10^{-1} sec. (Note that 1.18×10^{-1} sec is the time necessary for the bending wave of the tenth mode to reach the control position directly when the boundary condition is clamped-free.) After the time t_0 , Eqs.(19.2) and (20.2) are used. Moreover, in order to reduce the cost of this active vibration control, the active control will be stopped at $t_1=10.0$ and t is fixed after that. The control method is finally given as $(0 \leq t \leq 1.18 \times 10^{-1})$

$$m_c = c_m(t_0) \frac{\partial \theta}{\partial t} \tag{21}$$

$$[t_0 = 1.18 \times 10^{-1}],$$

$$(1.18 \times 10^{-1} < t < 10.0)$$

$$m_c = c_m(t) \frac{\partial \theta}{\partial t}$$

$$v_c = c_v(t) \frac{\partial w}{\partial t}, \tag{22}$$

$(t \geq 10.0)$

$$m_c = c_m(t_1) \frac{\partial \theta}{\partial t}$$

$$v_c = c_v(t_1) \frac{\partial w}{\partial t} \tag{23}$$

$$[t_1 = 10.0].$$

Figure 4 shows the deflection of the right end as the result of this method. The dotted and the solid

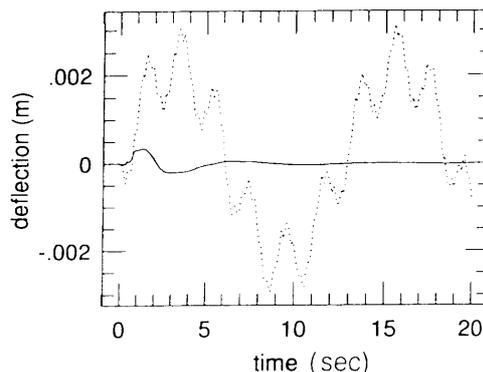


Fig. 4 Concentrated impulse response at the right end

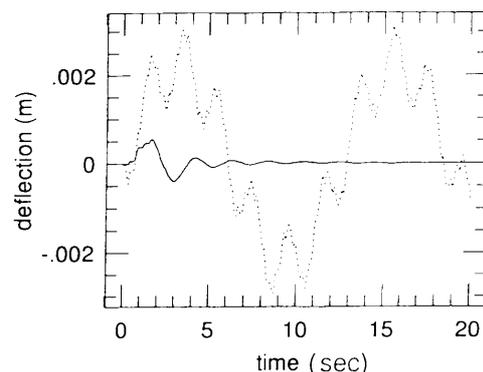


Fig. 5 Concentrated impulse response at the right end (only shear force control)

lines represent the uncontrolled and the controlled conditions, respectively. Sufficient restraint of the vibration is obtained.

This wave-absorbing control method is theoretically given by both Eq.(19) using the moment force and Eq.(20) using the shear force. But, in practice, both of these control forces may not always be used simultaneously. Noting that Eq.(19.2) is a function proportional to t and that Eq.(20.2) is a function inversely proportional to t , we find that the shear force works well as the control force when t is very small. Specifically, the shear force has the important effect of restraining the vibration for small t . We then consider the case when only the shear force is used for control, that is, the moment force is zero. Figure 5 shows the deflection for concentrated impulse load at the right end. Effective restraint is obtained, although it is slightly inferior to the result in Fig. 4.

4.2 Distributed impulse response

In the former section, the bending vibration was controlled when the load condition was a concentrated impulse. Here our method is applied to the case when the load condition is an arbitrary distributed load. Again, the control position is the right end.

We regard the distributed load as superposed

concentrated loads. The function $f(x)$ represents the distributed load on the interval $[x_1, x_2]$, which is divided into n intervals. For a wave which is generated at a point $x_r (= (x_1 + x_2 - x_1) \cdot n/r)$, the wave-absorbing control method similar to the former section is given by

$$m_c = \frac{EI}{2} t \left\{ \frac{1}{l-x_r} + \frac{1}{l+x_r} \right\} \frac{\partial \theta}{\partial t}, \quad (24)$$

$$v_c = -EI \frac{l}{a^2 t} \frac{\partial w}{\partial t}. \quad (25)$$

Summing the above equations about each x_r gives

$$\sum_{r=0}^n m_c = \sum_{r=0}^n \frac{EI}{2} t \left\{ \frac{1}{l-x_r} + \frac{1}{l+x_r} \right\} \frac{\partial \theta}{\partial t}, \quad (26)$$

$$\sum_{r=0}^n v_c = - \sum_{r=0}^n EI \frac{l}{a^2 t} \frac{\partial w}{\partial t}. \quad (27)$$

Then these equations yield

$$m_c = \frac{\sum_{r=0}^n \frac{EI}{2} t \left\{ \frac{1}{l-x_r} + \frac{1}{l+x_r} \right\} \frac{\partial \theta}{\partial t}}{\sum_{r=0}^n 1}, \quad (28)$$

$$v_c = \frac{- \sum_{r=0}^n EI \frac{l}{a^2 t} \frac{\partial w}{\partial t}}{\sum_{r=0}^n 1}. \quad (29)$$

When $n \rightarrow \infty$, the righthand sides of Eqs.(28) and (29) reduce to the following using the definition of the integral.

$$m_c = \tilde{c}_m(t) \frac{\partial \theta}{\partial t} \quad (30.1)$$

where

$$\tilde{c}_m(t) = \frac{EI t}{2(x_2 - x_1)} \ln \frac{(l-x_1)(l+x_2)}{(l-x_2)(l+x_1)}, \quad (30.2)$$

and

$$v_c = - \tilde{c}_v(t) \frac{\partial w}{\partial t} \quad (31.1)$$

where

$$\tilde{c}_v(t) = EI \frac{l}{a^2 t}. \quad (31.2)$$

These equations are stable because $\tilde{c}_m(t)$ and $\tilde{c}_v(t)$ are positive.

As an example, we examine the response for the case when the distributed impulse load forms a rectangular wave (the interval of which is $[1.35, 1.65]$). As in the former section, control is divided into three stages.

- $(0 \leq t \leq 0.1)$
Passive [$t_0=0.1$],
- $(0.1 < t < 10.0)$
Active
- $(t \geq 10.0)$
Passive [$t_0=10.0$].

Figure 6 shows the controlled deflection of the right end compared with the uncontrolled case. Sufficient restraint of the vibration is obtained.

Since the functions in Eqs.(30) and (31) are related to time t as in the former section, we consider

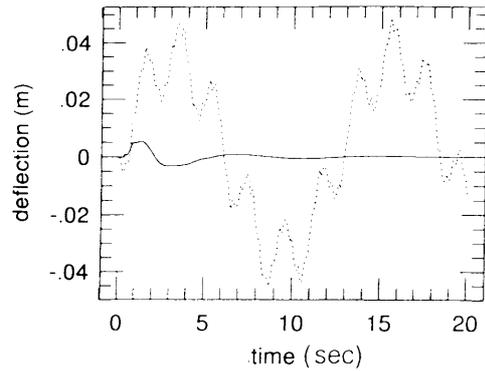


Fig. 6 Distributed impulse response at the right end

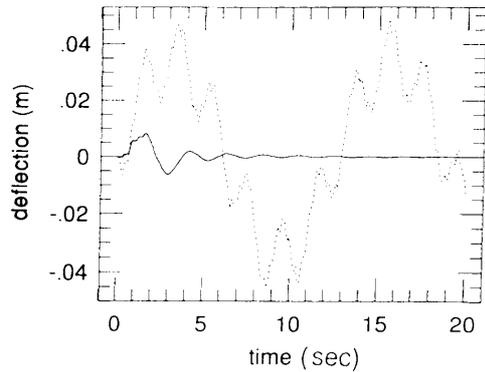


Fig. 7 Distributed impulse response at the right end (only shear force control)

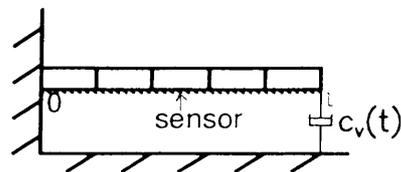


Fig. 8 Experimental device model

again the case when only the shear force is used for the control. Figure 7 shows that the vibration is again effectively restrained.

The control method which uses both the moment and the shear forces requires information about the position of the impulse load and the time of initial load in advance. In this way, it will be possible to know the information before the elastic wave propagates, for instance, by using the light fiber. However, effective restraint may be obtained using only the shear force. In this case the only initial time is required, so it is sufficient to locate a sensor at a suitable position of the beam and a dashpot for the shear force where its coefficient $c_v(t)$ may be changed as needed as shown in Fig. 8.

5. Conclusion

We propose a method of active vibration control for flexible structures using wave-absorbing control. It is based on the fact that standing waves generated in the structure play an important role in continuous vibration, so the formation of standing waves is actively restrained.

The Euler-Bernoulli beam is employed as the control object. The general solution is represented by a sine-wave which represents the forward and backward waves forming the standing waves. Then the wave-absorbing control method is obtained by eliminating the outgoing wave at the control position, namely, absorbing the incoming wave. To confirm the effectiveness of this control method, the response in frequency domains is examined. As a result, effective restraint is obtained in all frequency domains.

Next, this method is applied to the problem of the impulse load response which has many vibration modes in a wide frequency range. By taking the propagation of the impulse wave into consideration in the dispersive bending vibration, we obtain effective restraint quickly, before standing waves are formed.

Appendix 1. Consideration of Stability

Stability is considered under the control method in Eqs.(4) and (5). Here we represent the basic governing equation of the Euler-Bernoulli beam as follows:

$$\frac{\partial}{\partial t} \mathbf{u}(x, t) = \mathbf{F} - \mathbf{u}(x, t) \quad (t > 0, 0 \leq x \leq l) \quad (\text{A.1.1})$$

where

$$\mathbf{u}(x, t) = \begin{bmatrix} w(x, t) \\ \frac{\partial w(x, t)}{\partial t} \end{bmatrix}, \quad (\text{A.1.2})$$

$$\mathbf{F} = \begin{bmatrix} 0 & 1 \\ -a^2 \frac{\partial^4}{\partial x^4} & 0 \end{bmatrix} \quad (\text{A.1.3})$$

and the external force term is neglected.

The boundary condition is represented as follows: at $x=0$

$$w(0, t) = \frac{\partial w(0, t)}{\partial x} = 0 \quad (\text{A.2})$$

and, at $x=l$, Eqs.(4) and (5)

$$m_c = EI\mu \frac{\partial \theta}{\partial t}, \quad (4)$$

$$v_c = -EI\nu \frac{\partial w}{\partial t} \quad (5)$$

give

$$\frac{\partial^2 w(l, t)}{\partial x^2} + \mu \frac{\partial^2 w(l, t)}{\partial t \partial x} = 0, \quad (\text{A.3})$$

$$\frac{\partial^3 w(l, t)}{\partial x^3} + \nu \frac{\partial w(l, t)}{\partial t} = 0, \quad (\text{A.4})$$

Then

$$\mathbf{F}\mathbf{u} = 0. \quad (\text{A.5})$$

Equations (A.2), (A.3) and (A.4) obviously give

$$\mathbf{u} = 0, \quad (\text{A.6})$$

so the origin (the zero solution) becomes the equilibrium point. Here Eqs. (A.1), (A.2), (A.3) and (A.4) are assumed to have a unique solution for the given initial conditions around the origin.

The following function $\mathbf{E}_v(\|\mathbf{u}\|)$ is introduced:

$$\mathbf{E}_v(\|\mathbf{u}\|) = \int_0^l \left\{ \frac{EI}{2} \left(\frac{\partial^2 w}{\partial x^2} \right)^2 + \frac{\rho A}{2} \left(\frac{\partial w}{\partial t} \right)^2 \right\} dx \quad (\text{A.7})$$

$\mathbf{E}_v(\|\mathbf{u}\|)$ is clearly a positive-valued function. Then the time differentiation of $\mathbf{E}_v(\|\mathbf{u}\|)$ along the trajectory of Eqs.(A.1), (A.2), (A.3) and (A.4) is given by

$$\dot{\mathbf{E}}_v(\|\mathbf{u}\|) = -EI\mu \left(\frac{\partial^2 w(l+1)}{\partial t \partial x} \right)^2 - EI\nu \left(\frac{\partial w(l, t)}{\partial t} \right)^2. \quad (\text{A.8})$$

Therefore, if both $(EI)\mu$ and $(EI)\nu$ are positive, $\dot{\mathbf{E}}_v(\|\mathbf{u}\|)$ is negative. Since, then, $\mathbf{E}_v(\|\mathbf{u}\|)$ is found to be a Lyapunov function, the zero solution of the equilibrium point is satisfied with an asymptotic stability.

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