HIGH RESOLUTION OF MULTI-FREQUENCIES USING MULTILAYER NETWORKS TRAINED BY BACK-PROPAGATION ALGORITHM

Kazuyuki HARA
Graduate School of Nat. Sci. & Tech.,

Kenji NAKAYAMA
Faculty of Tech., Kanazawa Univ.

2–40–20 , Kodatuno , Kanazawa , 920 JAPAN
E–mal : hara@haspnn1.ec.t.kanazawa-u.ac.jp
nakayama@haspnn1.ec.t.kanazawa-u.ac.jp

ABSTRUCT Frequency analysis capability of multilayer neural networks trained by back-propagation (BP) algorithm is investigated. Multi-frequency signal classification is taken into account for this purpose. The number of frequency sets, that is signal groups, is 2∼5, and the number of frequencies included in a signal group is 3∼5. The frequencies are alternately located among the signal groups. Through computer simulation, it has been confirmed that the neural network can provide very high frequency resolution. Classification rates are about 99.5% for training signals, and 99.0% for untraining signals. The number of hidden units is 1∼8, which depends on the numbers of groups and frequencies. The classification results are compared with conventional methods. First, Euclidean distance is not useful. Accuracy is about 65% for some cases. Second, Fourier transform is also useless. When the observation interval is short, and the number of samples is limited, it cannot distinguish close located frequency groups. In some cases, accuracy is about 10∼30%. Separating each frequency by using very high-Q filters, is possible, even though the data samples are limited. However, it requires a very high filter order, resulting in a huge amount of computations. On the contrary, the neural network requires only the same number of inner products as the hidden units. Consequently, it can be concluded that the neural network can resolve multi-frequency with a small number of data samples, high accuracy and less computations.

I INTRODUCTION

Advantage of multilayer neural networks trained by back-propagation (BP) algorithm is to extract common properties, features or rules, which can be used to classify data included in several groups [1]. Especially, when it is difficult to analyze the common features using conventional methods, the supervised learning, using combinations of the known input and output data, becomes very useful. This application field includes, for instance, pronunciation of English text, speech recognition, image compression, sonar target analysis, stuck market prediction and so on [2]-[6].

In this paper, classification performance of the neural networks is discussed based on frequency analysis. Multi-frequency signals are employed for this purpose. Especially, we are interested in super-resolution, that is, a short observation interval and a small number of samples are assumed. Performances of multilayer neural networks based on very limited information are investigated. Furthermore, the results are compared with conventional methods, Euclidean distance, Fourier transform and filtering methods.

II MULTI–FREQUENCY SIGNALS

Multi–frequency signals are defined by

$$\begin{aligned} X_{pm}(n) &= \sum_{r=1}^{R} A_{mr}\sin(\omega_{pr}nT+\phi_{mr}) \\ n &= 0\sim N-1\ ,\ \omega_{pr}=2\pi f_{pr} \end{aligned} \tag{1}$$

T is a sampling period. R is the number of frequencies included in the same group. The signals have N samples. $X_{pm}(n), m=1\sim M$, are included in the group X_p as follows:

$$X_p=\{X_{pm}(n), m=1\sim M\}\ \ ,\ \ p=1\sim P \tag{2}$$

The pth set of multi-frequencies is expressed by

$$F_p = [f_{p_1}, f_{p_2}, \cdots, f_{p_R}]Hz \quad , \quad p = 1 \sim P \tag{3}$$

Amplitude A_{mr} and phase ϕ_{mr} are different for each frequency component. They are generated as random numbers, uniformly distributed in the following ranges.

$$0 \ < \ A_{mr} \leq 1 \tag{4}$$

$$0 \ \leq \ \phi_{mr} < 2\pi \tag{5}$$

III MULTILAYER NEURAL NETWORK

A two-layer neural network is taken into account. N samples of the signal $X_{pm}(n)$ are applied to the input layer in parallel. Thus the nth input unit receives the sample at nT. The number of output units is equal to that of the signal groups P. The neural network is trained by the BP algorithm[1] so that a single output unit responds to one of the signal groups.

3.1 Training and Classification

Multi-frequency signals in the pth group are devided into training and untraining data sets, X_{Tp} and X_{Up} , respectively. Their elements are expressed by $X_{Tp_m}(n)$ and $X_{Up_m}(n)$ as follows:

$$X_p \ = \ [X_{Tp}, X_{Up}] \tag{6}$$

$$X_{Tp} \ = \ \{X_{Tp_m(n)}, m = 1 \sim M_T\} \tag{7}$$

$$X_{Up} \ = \ \{X_{Up_m(n)}, m = 1 \sim M_U\} \tag{8}$$

The neural network is trained by using $X_{Tp_m}(n)$ for the pth group. After the training completes for all frequency groups, the untraining signals $X_{Up_m}(n)$ are applied to the nueral network. If the kth output unit has the maximum input, then the signal is classified into the kth group. Therefore, when $X_{Up_m}(n)$ is applied, if k = p, then the signal is exactly classified, and otherwise the classification is not successful.

IV MULTI–FREQUENCY SIGNAL CLASSIFICATION

4.1 Multi-Frequency Signals

Six kinds of multi-frequency signal sets are used as shown in Table 1. The number of frequencies is 3 ∼ 5 and that of signal groups is 2 ∼ 5, respectively. In Case-1.2, the frequencies in both groups are very close. In Case–2.2 and 3.3, the number of samples is N = 15 and N = 20, the sampling period is T = 1/15sec and 0.05sec, respectively. In the other cases, the sampling frequency is 10 Hz, that is T = 0.1sec. The number of samples is N = 10. Therefore, the observation interval is 1 sec in all cases.

4.2 Neural Network Classification

Table 1 illustrates simultion results. The training data set of each group include 200 signals, that is $X_{Tp_m}(n)$, $m = 1 \sim 200$. 1800 signals are used as untraining signals in each group. Namely, $X_{Up_m}(n)$, $m = 1 \sim 1800$.

In Case–1.1, 1.2, 2.2, training converged with one hidden unit. A accuracy for X_{Tp} is, therefore, 100 %. For the untraining signals, the classification rate is around 99 %. In the other cases, the training did not completely converge. However, classification accuracy is also very high, that is about 99.5 %. Thus, highly exact classification can be achieved.

Table 1 Classification results by neural networks [%]

CASE	Frequency Sets [Hz]	Hidden Units	Accuracy %	
			X_{TP}	X_{UP}
1.1	F_1 = [1,2,3]	1	100	99.8
	F_2 = [1.5,2.5,3.5]		100	94.5
1.2	F_1 = [1,2,3]	1	100	99.7
	F_2 = [1.1,2.1,3.1]		100	99.7
2.1	F_1 = [1,1.5,2,2.5,3]	5	82.8	77.2
	F_2 = [1.25,1.75,2.25,2.75,3.25]		86.8	82.3
2.2	F_1 = [1,2,3,4,5]	1	100	99.9
	F_2 = [1.5,2.5,3.5,4.5,5.5]		100	99.9
3.1	F_1 = [1,2,3]	4	99.0	93.5
	F_2 = [1.33,2.33,3.33]		99.5	98.8
	F_3 = [1.67,2.67,3.67]		100	99.6
3.2	F_1 = [1,2,3]	8	98.0	96.3
	F_2 = [1.2,2.2,3.2]		79.5	75.3
	F_3 = [1.4,2.4,3.4]		98.5	98.1
	F_4 = [1.6,2.6,3.6]		89.0	87.2
	F_5 = [1.8,2.8,3.8]		98.0	97.8
3.3	F_1 = [1,4,7]	8	99.5	98.4
	F_2 = [1.5,4.5,7.5]		99.5	98.5
	F_3 = [2,5,8]		99.0	99.1
	F_4 = [2.5,5.5,8.5]		99.0	97.2
	F_5 = [3,6,9]		100	99.6

4.3 Euclidean Distance Analysis

Some aspect of similarity between the training and untraining signals can be evaluated using Euclidean distance defined by

$$D_{pq}(m,m') = \{\frac{1}{N}\sum_{n=1}^{N}(X_{T p_m}(n) - X_{U q_{m'}}(n))^2\}^{\frac{1}{2}} \tag{9}$$

In this section, classification based on this distance is investigated. Apply a multi-frequency signal $X_{Up_m}(n)$, and calculate the Euclidean distance between $X_{Up_m}(n)$ and all other training signals. If the training signal, having the minimum Euclidean distance, is included in X_{Tq}, then $X_{p_m}(n)$ is classified to the qth group.

In Case–1.1 and 1.2, accuracy is 98 % and 61.3 %, respectively. Thus, the Euclidean distance method is impractical to distinguish the frequencies locate close each other, such as Case–1.2.

4.4 Fourier Transform Analysis

Fourier transform of a descrete-time signal $X_{p_m}(n)$ is given by

$$G_{p_m}(e^{j\omega T}) = \sum_{n=0}^{N-1} X_{p_m}(n)e^{-j\omega nT} \tag{10}$$

Classification is carried out as follows: Let $|G_{p_m}(e^{j\omega T})|$ be $A_{p_m}(f)$ for convenient.

Rule1: If $A_{p_m}(f_{p_i}) > A_{p_m}(f_{q_i})$ for all q($\neq p$) and i = 1~R, then $X_{p_m}(n)$ is classified into the pth group.

Rule2: If $A_{p_m}(f_{p_i}) > A_{p_m}(f_{q_i})$ at more than half of i = 1 ~ R for all q, then $X_{p_m}(n)$ is classified to the pth group.

Figure 1 shows examples for amplitude responses of $X_{1m}(n)$ in Case–1.1 and 3.3. Since in Case–1.1, the sampling frequency is set to 10Hz, an observable frequency range is up to 5Hz. $X_{1m}(n)$ includes multi-frequency of [1 , 2 , 3]Hz, amplitude responses at these frequecies are not always greater than those at frequencies of [1.5 , 2.5 , 3.5]Hz. Reasons are a short observation period and a small number of samples. In Case–3.3, classification performance is improved due to the increased number of samples from 10 to 20.

Table 3 shows persentage of exact classification by Fourier transform analysis based on Rule 1 and Rule 2. Accuracy is very low compared with the neural network version.

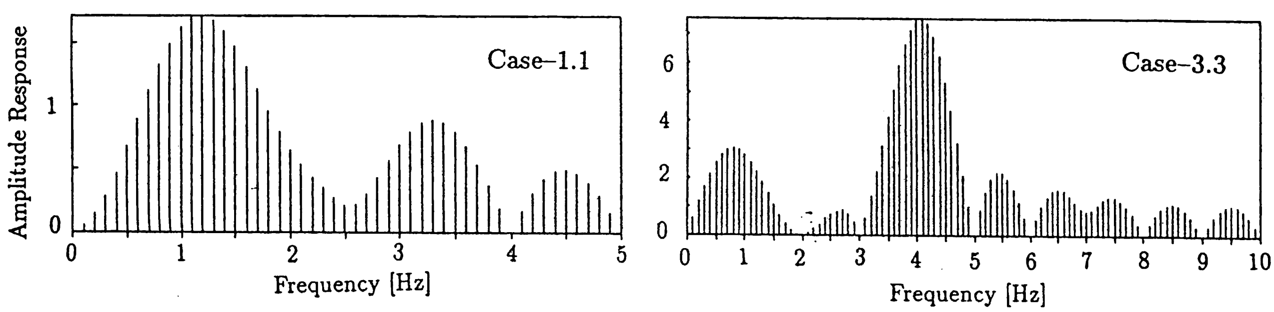

Fig.1 Examples of amplitude responses of $X_{1m}(n)$.

Next, ability of Fourier transform is further investigated by increasing the number of samples. This method requires about 60 samples and 100 samples for Case–1.1 and 1.2, respectively, in order to achieve the same resolution as the neural network. Thus, efficiency of the neural network is apparent.

Consequently, classification ability of the Fourier transform is highly dependent on the number of samples and the observation interval. Furthermore, it requires complex coefficients as shown in Eq.(10).

4.5 Filter Analysis

Frequency component extraction is also possible using digital filters with real coefficients[7].

Figure 3 shows a block diagram for frequency analysis using filters. Filters H_1 and H_2 have very sharp frequency selectivity at F_1 and F_2, respectively. This means very higt-Q filters are required. The multi-frequency signal is applied to both H_1, H_2, their frequency components are extracted by H_1 and H_2. The mean square of the output signals from the H_1 and H_2 filters are compared with each other. In order to achieve very high frequency resolution, very high-Q filters are required. In this simulation, a very long impulse response having 4000 samples was used. Accuracy is 100% for all cases.

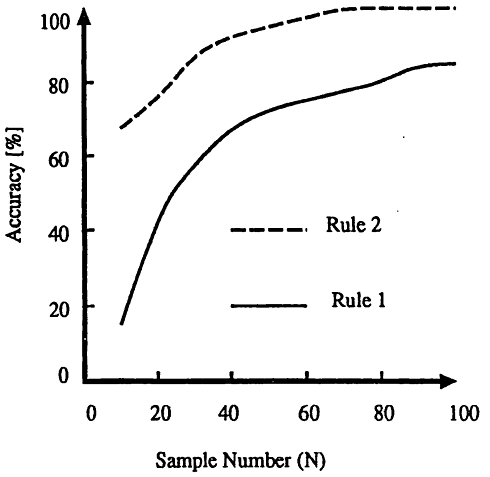

Fig. 2 Sample number and accuracy of Fourier transform.

Table 3 Classification results by neural network and Fourier transform analysis.

CASE	Hidden unit	group	Neural Net Accuracy %		Fourier Trans Accuracy %	
			X_{Tp}	X_{Up}	Rule1	Rule2
1.1	1	1	100	99.8	11.3	82.6
		2	100	94.5	12.9	81.1
1.2	1	1	100	99.7	4.1	50.0
		2	100	99.9	16.6	77.7
2.1	5	1	82.8	77.2	0.2	10.0
		2	86.8	82.3	0.4	12.3
2.2	1	1	100	99.9	0.9	17.7
		2	100	99.9	1.2	19.2
3.1	4	1	99.9	93.5	0.0	14.1
		2	99.5	98.8	2.9	36.7
		3	100	99.6	8.6	64.3
3.2	8	1	98.0	96.3	0.8	27.0
		2	79.5	75.3	0.0	10.4
		3	98.0	98.1	0.0	6.5
		4	89.0	87.2	0.0	5.0
		5	98.0	97.8	0.0	28.4
3.3	8	1	99.5	98.4	60.2	97.1
		2	99.5	98.5	47.9	94.9
		3	99.5	98.4	49.0	90.8
		4	99.0	97.2	47.8	94.3
		5	100	99.6	56.5	96.9

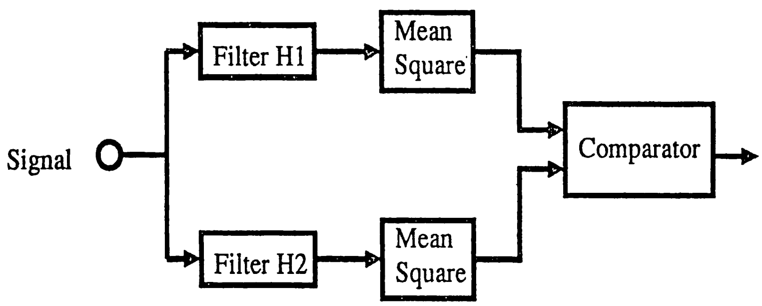

Fig. 3 Block diagram of classification system for two frequency groups.

The neural network requires only the same number of inner products of real vectors as the hidden units. On the contrary, the filter method needs a hudge amount of computations.

V CONCLUSIONS

Frequency resolution capability of the neural network has been discussed. Comparing conventional methods, Euclidean distance, Fourier transform and filtering methods, the multilayer neural network is very superior to them in the following points. First, the neural network can resolve the multi-frequency signals with very high accuracy using limited information. Second, computational requirements are very small.

References

[1] D.E.Rumelhart and J.L.McClelland et al, Parallel Distributed Processing,MIT Press, 1986.

[2] T.J.Sejnowski and C.R.Rosenberg,"Parallel networks that learn to pronounce English text ",Complex Systems,vol.1, pp.145-168, 1987.

[3] J.L.Elman and Zipser,"Learning the hidden structure of speech",Tech. Rep. 8701, Univ. of California, San Diego, 1987.

[4] G.W.Cottrell,R.Munro and D.Zipser,"Image compression by backpropagation: An example of extention programming", Tech. Rep. 8702, Univ. of California, San Diego, 1987.

[5] R.P.Gorman and T.J.Sejnowski,"Analysis of hidden units in a layered network trained to classify soner targets", Neural Networks, vol.1,pp.75–90, 1988.

[6] T.Kimoto et al.,"Stuck market prediction system with modular neural netwoeks", Proc. IJCNN'90, San Diego, pp.I-1–6,1990.

[7] A.V.Oppenheim and R.W.Shafer,Digital Signal Processing, Prentice–Hall, Inc., 1975.

[8] K.Hara and K.Nakayama,"Multi–frequency signal classification using multilayer neural network trained by backpropagaation algorithm",Tech. Rep. IEICE, NC–75, pp.47–54, Nov. 1992.