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Abstract— In this paper, we examine the H_∞ Filter-based SLAM especially about its convergence properties. In contrast to Kalman Filter approach that considers zero mean gaussian noise, H_∞ Filter is more robust and may provide sufficient solutions for SLAM in an environment with unknown statistical behavior. Due to this advantage, H_∞ Filter is proposed in this paper, to efficiently estimate the robot and landmarks location under worst case situations. H_∞ Filter requires the designer to appropriately choose the noise's covariance with respect to γ to obtain a desired outcome. We show some of the conditions to be satisfy in order to achieve better estimation results than Kalman Filter. From the experimental results, H_∞ Filter performs better than Kalman Filter for a case of bigger robot initial uncertainties. Subsequently, this proved that H_∞ Filter can provide another available estimation method for especially in SLAM.

Fig. 1.   Illustration for SLAM problem

I. INTRODUCTION

A. Robotic Mapping

Robotics localization and mapping problem is one of the autonomous robot applications that recently gained researcher's attention thanks to its capability that able to support autonomous robot behavior. The problem illustrates a case where a mobile robot is put in an unknown environment, then takes sufficient observations about its surroundings. Next, from this information, robot then builds a map from what it believes. Even though the development of the robot localization and mapping problem has passed about two decades, there are still a lot of difficulties to be solved.

Since 1990's, researchers around the world become more passionate about this problem and a series of convincing seminal papers by Smith and Cheeseman et.al [1], has urged up this research. Consequently, its name is evolving to Simultaneous Localization and Mapping problem(SLAM)[2]. See Fig.1 for SLAM illustration. As stated by its name, SLAM consists of two general problems that are the robot localization and mapping. Robot localization states a problem where we are given predefined landmarks, the robot must attempt to estimate it location relative to the map. While, robot mapping determines a problem that given a robot trajectory, a map must be built. Therefore, SLAM is more complicated and needs proper effort for the solution.

Nowadays, SLAM has been applied in a wide application, indoor or outdoor such as satellite, min-
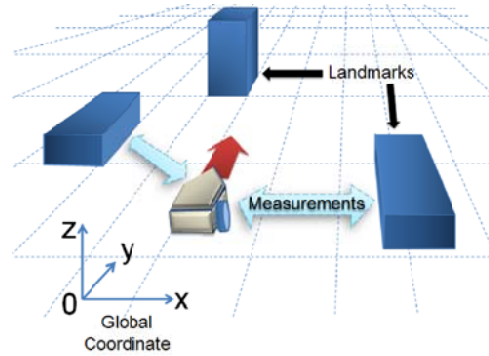
ing, space exploration, rescue, military, etc. SLAM research progress in 2D[3] or 3D applications [4][5] and amazingly expand even to home-based robot application. The problem is tracked in 1980's, and improved from the form of *Topological* and *Metric* approaches to *Behavioral approach*, *Mathematical-based model* approach and *Probabilistic approach* [2]. However, between these 3 techniques, the probabilistic approach made a significant success than the mathematical models approach; which require building a precise model, or the behavior approach; a method of exploiting the sensor's behavior to the system. In spite of remarkable achievement of probabilistic approach, it has a shortcoming of computational complexity. Nevertheless, with modern software development, a considerable support and solution to this problem may be available, and thus inspired the development of SLAM problem.

Recently, probabilistic approaches, whether parametric or non-parametric methods have been proposed to solve the SLAM problems such as Kalman Filter, Unscented Kalman Filter, Particle filter, etc.. At this end, a non-parametric method so-called Fast-SLAM approach [2], which is claimed can efficiently constructs the unknown map by utilizing an amount of particle whose behaves as the uncertainty. If more particles are used, the estimation will be better, but in contrast they require a high computational cost for the systems. Due to such deficiencies, such a impressive technique does not intimidate classical methods, for example, Kalman Filter. Moreover, no matter what kind of filters presented above, they are familiar and fundamentally relied on probabilistic theory. The readers are encouraged to read about the development of SLAM in [6] which discussed the SLAM problem from various aspects.

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B. Probabilistic-based SLAM

Uncertainties and sensor noises are the most influential elements that brought the idea of probabilistic into SLAM problem. Governed by the law of probabilistic, the estimation is processed to a set of information than only rely on a single guessed method. This eventually made probabilistic method applicable to most SLAM problems in most situations with unknown noise characteristics.

In contrast to Kalman Filter reputation among decades within various fields, some applications still demands further attention for development especially due its deficiencies of zero mean gaussian assumption. In fact, it is a wise decision to model a system that takes into account for a worst case of noise or when the noise statistics are partially known. Hence, H_∞ Filter is proposed in this paper, to tolerate such a robust system. The development of H_∞ Filter for SLAM problem is theoretically shown with a brief comparison to Kalman Filter approach[7][8]. H_∞ Filter[9] is one of the set-membership approaches, which assumes that the noise is known in bounded energy. It is also a technique that assumed the systems are provided with a priori information for estimation[10][11]. H_∞ Filter(HF) guarantees that the energy gain from the noise inputs to the estimation errors is less than a certain level. A study on its convergence has been proposed by Hamzah et.al[12]. Further application of *Covariance Inflation* to decrease the computation cost and avoid *Finite Escape Time* in HF also studied in [13].

Throughout this paper, we examine the Kalman Filter and HF performance in linear and nonlinear case SLAM problem. We investigate the results using a constant motion and sensors uncertainties with a perfect data association. Even though this is seems to be simplistic, it gives a feasible study about the estimation. HF has desirable properties and is competitive compared to Kalman Filter[14][15] especially in SLAM. [14] reported that, EKF with robocentric local mapping approach, is able to decrease location uncertainty of each location. West et.al[17] proved that HF was competent with other well-known approaches such as KF and Particle filter for SLAM problem. However, they did not present any theoretical explanation or contribution about HF properties. We show that an appropriate selection between the γ, initial state covariance, process and measurement noise covariances enable HF to perform better in SLAM than KF.

This paper is organized as follows. In *Section II*, SLAM preliminary model is presented. *Section III* describes a brief introduction of HF with a comparison between HF algorithm and the KF, while *Section IV* demonstrates the main results of convergence properties of H_∞ SLAM. *Section V* represents the experimental results of SLAM using both filters. Finally, *Section VI*, concludes the paper.

II. SLAM Preliminary Model

SLAM consists of two general models; Process Model and Measurement/Observation Model(see Fig.2). Each of this model plays an important role to achieve better estimation about the landmarks and robot location. For the SLAM process model, we have the following. We consider the linear
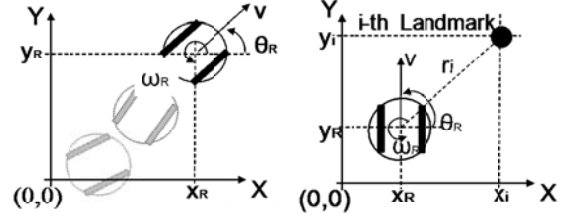

Fig. 2. SLAM Process and measurement model

SLAM problem as most of the calculation is linearized in the entire process and may sufficiently describes the whole system.

$$x_{R_{k+1}} = F_{R_k} x_{R_k} + u_{R_k} + v_{R_k}, \tag{1}$$

From above, F_{R_k} is the state transition matrix, $x_{R_k} \in \mathbb{R}^3$, is the robot state, $u_{R_k} \in \mathbb{R}^2$ is a vector of control inputs, and v_{R_k} is a vector of temporally uncorrelated process noise errors with zero mean and covariance, Q_{R_k}. The location of the nth landmark is denoted as p_n and the landmarks are assumed to be stationary. The stationary landmarks states are expressed by

$$p_{n_{k+1}} = p_{n_k} = p_n \tag{2}$$

where $n = 1 \ldots N$. Using above notation with respect to [1], the augmented process model consists of robot and landmarks location is described as following.

$$x_{k+1} = F_k x_k + u_k + v_k \tag{3}$$

x_k is the augmented state, while $u_k = u_{R_k}$ and $v_k = v_{R_k}$ as landmarks is stationary. On the other hand, the measurement model includes information about relative distance and angle between the robot and any landmarks. The observation at i_{th} specific landmarks, yields the following equation.

$$
\begin{aligned}
z_k &= H_k x_k + w_k \tag{4} \\
&= H_{p_i} p_i - H_{v_k} x_{(v_k)} + w_k \tag{5}
\end{aligned}
$$

where w_k is a vector of temporally uncorrelated observation errors with zero mean and variance R_k. H_k is the observation matrix and represents the output of the sensor z_k to the state vector x_k when observing the ith landmark. $H_{p_i} - H_{v_k}$ is the relative measurement matrix between the landmarks and the robot respectively. Alternatively, the observation model for the ith landmark is written in the form

$$H_k = [-H_v, 0 \ldots 0, H_{pi}, 0 \ldots 0] \tag{6}$$

Above equation shows observations are taken as a relative measurement between vehicle and landmarks. Both models are used recursively to predict and updates both landmarks and robot position. Based on the data obtained from these two models, then the robot built a map. Same to KF, HF has the prediction and updates process. Details are explained in the next section consisting of some basic assumption of noises and a brief comparison to the KF approach.

III. H_∞ FILTER-BASED SLAM

This section presents the development of HF-Based SLAM by considering its convergence properties. Due to our approach is a probabilistic SLAM, the state covariance matrix plays an important role to determine the level of confidence for estimation. In SLAM, small state covariance matrix is desired. Hence, the analysis is focusing on the convergence behavior of HF-Based SLAM, whether it may surpass KF performance or else.

The comparison between HF and KF for a stationary robot case observing landmarks is evaluated in the experiments. Some brief explanation and preparation are introduced regarding the differences between both filters before getting in depth with the filter performance in SLAM. The papers in [7][9] presented a satisfactory explanation for HF. Referring to those, first we assumes that the noises hold the following properties.

Assumption 1: $R_k \overset{\Delta}{=} D_k D_k^T \geq 0$

The above assumption is used to define that the measurements are correlated with noise. We further assume that the noises is in bounded energy which also a characteristic of HF. This is the main dissimilarity between HF and KF.

Assumption 2: Bounded noise energy; $\sum_{t=0}^{N} \|w_k\|^2 < \infty, \sum_{t=0}^{N} \|v_k\|^2 < \infty$
$Q_k \geq 0$, and $R_k \geq 0$ are the weighting matrices for process noise w_k, and measurement noise v_k respectively[7].

The differences between KF and HF exists in the form of gain and covariance characteristics for each prediction and updates process. For KF gain K_k and its covariance P_k we have the following.

$$K_k = P_k(I + H_k^T R_k^{-1} H_k P_k)^{-1} \tag{7}$$
$$P_{k+1} = F_k P_k (I + H_k^T R_k^{-1} H_k P_k)^{-1} F_k^T + Q_k \tag{8}$$

As for HF, the equation for its gain K_k and covariance P_k are given by

$$K_k = P_k(I - \gamma^{-2} I P_k + H_k^T R_k^{-1} H_k P_k)^{-1} \tag{9}$$
$$P_{k+1} = F_k P_k (I - \gamma^{-2} I P_k + H_k^T R_k^{-1} H_k P_k)^{-1} F_k^T + Q_k \tag{10}$$

I is an identity matrix with an appropriate dimension and the other variables has been explained before. Stated above, HF depends on the covariance matrix of error signals, $Q_k \geq 0, R_k \geq 0$ which are chosen and designed to achieve a desired performance. It is observable that, if γ values become bigger, these equation approximating (7),(8) of KF.

IV. MAIN RESULTS

We begin the convergence analysis of HF by presenting the filter algorithm as stated below. The solution of an HF problem is as following[7]. Given that $P_0 \geq 0$.

$$P_{k+1} = F_k P_k \psi_k^{-1} F_k^T + G_k Q_k G_k^T, \quad P_0 = \Sigma_0 \tag{11}$$
$$\psi_k = I + (H_k^T R_k^{-1} H_k - \gamma^{-2} I) P_k \tag{12}$$

where I is an identity matrix with an appropriate dimension. Equation (11), (12) holds a Positive Semidefinite(PsD) solution if it satisfies

$$\hat{P}_k^{-1} - \gamma^{-2} I \geq 0, \quad k = 0, 1, \ldots, N, \tag{13}$$

where

$$\hat{P}_k = (P_k^{-1} - H_k^T R_k^{-1} H_k) \geq 0 \tag{14}$$

For $\gamma > 0$, the suboptimal HF is given by below equations.

$$\hat{x}_{k+1} = F_k \hat{x}_k \tag{15}$$
$$\hat{x}_k = \hat{x}_{k-1} + K_k[y_k - H_k \hat{x}_{k-1}], \hat{x}_{0|-1} = \bar{x}_0 \tag{16}$$
$$K_k = P_k H_k (H_k P_k H_k^T + R_k)^{-1} \tag{17}$$

From above, $\hat{x}_k$ is the estimated state, and $\bar{x}_0$ is the initial state with an initial state covariance, P_0.

Assumption 3: (F_k, H_k) is observable and (F_k, G_k) is controllable.

Lemma 1: For $P_0 \geq 0$, (11) is a PsD matrix if and only if $R_k \leq \gamma^2$.

Proof: For convenience, a 1-D monobot, a robot with a single coordinate system, observing one landmark case is considered. Given that the initial covariance matrix, P_0

$$P_0 = \begin{bmatrix} P_R & 0 \\ 0 & P_m \end{bmatrix} \tag{18}$$

where P_R is the monobot state covariance and P_m is a landmark state covariance. If $\gamma^2 \geq R_k$ then (12) always exhibit a PsD matrix. This can be proven as follows. Note that for 1-D monobot case, the measurement matrix becomes $H = \begin{bmatrix} -1 & 1 \end{bmatrix}$.

$$H_k^T R_k^{-1} H_k - \gamma^{-2} I = \begin{bmatrix} R_k^{-1} - \gamma^{-2} & R_k^{-1} \\ R_k^{-1} & R_k^{-1} - \gamma^{-2} \end{bmatrix}$$
$$\geq 0 \tag{19}$$

If else, (19) exhibit negative definite matrix or indefinite matrix and therefore causing unreliable estimation to HF. ∎

Even though *Lemma 1* is simplistic and illustrates the results of a monobot case, these result can reasonably aid the analysis for more complex system of 2D and 3D systems due to (11), (12) are acting as the main algorithm for HF. This is proven in the experimental results that are shown in later section. We proposed some other conditions for HF in SLAM in the following theorem.

Theorem 1: Assume that Assumptions 1~2 are satisfied. For $\gamma > 0$, the map uncertainties are gradually decrease if the following conditions are satisfied.

1) Equation (14) is also a PsD if the the measurement covariance noise, R is bigger than the state covariance matrix, P
2) γ^{-2} must be less than (14)
3) $H_k^T R_k^{-1} H_k - \gamma^{-2} I \geq 0$
4) *Lemma 1* is satisfied

Else, the state covariance matrix is not decreasing or may exhibit *Finite Escape Time* shown by[13].

Proof: Let the initial state covariance matrix, $P_0 \geq 0$, the process noise, $Q_k \geq 0$ and the measurement noise, $R_k \geq 0$. To ensure the state covariance matrix converge, there are some conditions to be satisfied. First, for $\gamma > 0$, (14) is also a PsD if the measurement covariance noise, R is bigger than the state covariance matrix, P. Second, in order to realize (13), γ^{-2} must be less than (14). Next, it is understood that, if

previous condition is satisfied, then $H_k^T R_k^{-1} H_k - \gamma^{-2} I \geq 0$. Finally for $\gamma^2 > R_k$, (12) result in PsD matrix. These four conditions must be fulfilled to achieve convergence of the state covariance matrix. If those conditions are satisfied, then from (10), the state covariance matrix P, yield the following. Let $W_k = H_k^T R_k^{-1} H_k - \gamma^{-2} I \geq 0$.

$$P_{k+1} = [P_k^{-1} + W_k]^{-1} \geq 0 \tag{20}$$
$$P_{k+2} = [P_{k+1}^{-1} + W_{k+1}]^{-1} \tag{21}$$
$$= [[P_k^{-1} + W_k]^{-1} + W_{k+1}]^{-1} \tag{22}$$
$$\leq P_{k+1} \tag{23}$$

From the PsD properties, any submatrix of a PsD is also a PsD. Hence, the submatrix of the landmarks components also hold the same characteristics.

$$P_{k+1_{mm}} \leq P_{k_{mm}} \tag{24}$$

We also found that for a case of the observation noise, $R >> \gamma$, the state covariance matrix exhibit negative definite matrix or escape in finite time[13]. Therefore, it may result in unstable estimations. ∎

It is also obvious that for a case of stationary landmarks, there is no process noise incorporated in the landmarks state's estimation. Thus, all the landmark covariance is expected to be constant through the observations. In other words, it is expected theoretically in the limit, the landmark covariance yield

$$P_{k+1_{mm}} \approx P_{k_{mm}} \tag{25}$$

Unfortunately, we show that this is not actually describes for the whole state covariance matrix in the next theorem.

State covariance matrix, P_k is generally a representation of uncertainties for each state estimation. [3] proposed some convergence properties for KF-Based SLAM. The results are then analyzed further in the nonlinear system by [18]. For HF in linear case SLAM, the convergence properties of a stationary robot observing landmarks are shown by [12]. We analyze further those results in SLAM.

Theorem 2: Suppose that *Theorem 1* is satisfied. For a stationary robot observing a stationary landmark m, with $\gamma > 0$, as more n-times$(n > 0)$ observation is made, in the limit, the whole covariance matrix is converging to

$$P_{k+1}^n = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \tag{26}$$

where

$$P_{11} = [P_{vv}^{-1} + n(R^{-1} - \gamma^{-2}I) - nR^{-1}(R^{-1} - \gamma^{-2}I)^{-1}R^{-1}]^{-1}$$
$$P_{12} = -P_{11}R^{-1}(R^{-1} - \gamma^{-2}I)^{-1}$$
$$P_{21} = -(R^{-1} - \gamma^{-2}I)^{-1}R^{-1}P_{11}$$
$$P_{22} = (R^{-1} - \gamma^{-2}I)^{-1}$$
$$+ (R^{-1} - \gamma^{-2}I)^{-1}R^{-1}P_{11}R^{-1}(R^{-1} - \gamma^{-2}I)^{-1}$$

If P_{11} exhibit a PsD, then the whole state covariance is decreasing. If else, the estimation is faulty or exhibit *Finite Escape Time*.

Proof: We consider a 2D robot with initial covariance matrix P_0.

$$P_0 = \begin{bmatrix} P_{vv} & 0 \\ 0 & P_{mm} \end{bmatrix} \tag{27}$$

$P_{vv} \in \mathbb{R}^3$ and $P_{mm} \in \mathbb{R}^2$ are the robot and landmarks initial covariance respectively. Assume that the robot is observing one landmark m at a certain point. From (18), when the stationary robot is observing m landmarks n times, we obtained the following equations.

$$P_{k+1}^{-1} = P_0^{-1} + n(H_{k_m}^T R_{k_m}^{-1} H_{k_m} - \gamma^{-2}I) \tag{28}$$

$$= P_0^{-1} + n \begin{bmatrix} R_{k_m}^{-1} - \gamma^{-2}I & R_{k_m}^{-1} \\ R_{k_m}^{-1} & R_{k_m}^{-1} - \gamma^{-2}I \end{bmatrix} \tag{29}$$

Assume that the initial state covariance matrix for the landmarks is very big. Then above equation yields

$$P_{k+1}^{-1} = \begin{bmatrix} P_{vv}^{-1} & 0 \\ 0 & P_{mm}^{-1} \end{bmatrix} + n \begin{bmatrix} R_{k_m}^{-1} - \gamma^{-2}I & R_{k_m}^{-1} \\ R_{k_m}^{-1} & R_{k_m}^{-1} - \gamma^{-2}I \end{bmatrix} \tag{30}$$

Finding the inverse matrix of (30) using the Matrix Inversion Lemma, yields

$$P_{k+1} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \tag{31}$$

where
$$P_{11} = [P_{vv}^{-1} + n(R_{k_m}^{-1} - \gamma^{-2}I) - nR_{k_m}^{-1}(R_{k_m}^{-1} - \gamma^{-2}I)^{-1}R_{k_m}^{-1}]^{-1}$$
$$P_{12} = -P_{11}R_{k_m}^{-1}(R_{k_m}^{-1} - \gamma^{-2}I)^{-1}$$
$$P_{21} = -(R_{k_m}^{-1} - \gamma^{-2}I)^{-1}R^{-1}P_{11}$$
$$P_{22} = (R_{k_m}^{-1} - \gamma^{-2}I)^{-1}$$
$$+ (R_{k_m}^{-1} - \gamma^{-2}I)^{-1}R_{k_m}^{-1}P_{11}R_{k_m}^{-1}(R_{k_m}^{-1} - \gamma^{-2}I)^{-1}$$

As long as $R_{k_m}^{-1} - \gamma^{-2}I \geq 0$, (29) is a PsD. Furthermore,
$$R_{k_m}^{-1} - \gamma^{-2}I \geq R_{k_m}^{-1}(R_{k_m}^{-1} - \gamma^{-2}I)^{-1}R_{k_m}^{-1}$$
$$R_{k_m}(R_{k_m}^{-1} - \gamma^{-2}I)R \geq (R_{k_m}^{-1} - \gamma^{-2}I)^{-1}$$

Above equation can be verified using PsD properties. Furthermore, from (30) and *Lemma 1*, it can be notice that for a case of the observation noise $R >> \gamma^2$, the state covariance matrix may suddenly increase and becomes unbounded. This is an unexpected behavior in SLAM. Thus, designer must properly choose an appropriate value to satisfy the proposed condition and achieve better estimation in HF. ∎

For the KF case, the state covariance is given by

$$P_{k+1}^{-1} = P_0^{-1} + n(H_k^T R_k^{-1} H_k) \tag{32}$$

$$= P_0^{-1} + n \begin{bmatrix} R^{-1} & R^{-1} \\ R^{-1} & R^{-1} \end{bmatrix} \tag{33}$$

Inverting the above matrix yield

$$P_{k+1}^n = \begin{bmatrix} P_{vv} & -P_{vv} \\ -P_{vv} & R + P_{vv} \end{bmatrix} \tag{34}$$

Observing the fact obtained by *Theorem 1*, it implicitly determines that the state covariance matrix for HF is slightly bigger than KF. The second and third variables on the right hand of (28), show explicitly the increment of the HF state covariance matrix. Furthermore, we inspect that (31)[12] may be a result of *Finite Escape Time*. To avoid this, the

TABLE I
EXPERIMENTAL PARAMETERS

γ	9
Process noise distribution, Q	$1e^{-6}$
Observation noise distribution, R	$\begin{vmatrix} 0.001 * I_2 \end{vmatrix}$
Random noise observation,R	$\begin{bmatrix} R_{\theta_{max}} = 0.05 \\ R_{\theta_{min}} = -0.05 \\ R_{distance_{max}} = 0.2 \\ R_{distance_{min}} = -0.2 \end{bmatrix}$
Initial Covariance(Case 1)Pvv, Pmm	$1e^{-5}$, 10000
Initial Covariance(Case 2)Pvv, Pmm	$1e^{-4}$, 0.001

Fig. 3.   Indoor experimental environment

Fig. 4.   Constructed map under zero mean Gaussian noises

conditions shown in *Theorem 1* required the designer to choose an appropriate noise energy level and γ to achieve desired performance. Furthermore, $R^{-1} - \gamma^{-2}I$ formulate how actually γ attempt to reduce the noise effect to the system. Besides, it proves that in the limit, the landmark state covariance is larger than the initial condition. Furthermore, it can be explicitly observed that from (28), if a process noise is increased, then the γ values also must be increase. Moreover, this properties can be extend for more landmarks in a case of a single robot observations.

V. EXPERIMENTAL RESULTS

This section evaluates the proposed theorems in an indoor environment. The experimental environment is shown in Fig.3, which consists of an E-puck robot with limited range sensing with some available landmarks in a small environment. E-puck movement and its observations about the landmarks between KF and HF are analyzed by Matlab via dSpace. We use a camera sensor as a virtual sensor to evaluate the performance between HF and KF. The parameters are chosen such that can satisfy *Theorem 1* for each P_0, Q_k, R_k and γ. The selection can be best obtain by first satisfying $\gamma >> R_k$ and then followed by the other variables. We made an assumption as stated below for the experiment to ensure that the characteristics and consistency are proved the convergence theorems.

Assumption 4: Robot is in planar world. Moreover landmarks stationary and consists of point landmarks

For the zero mean gaussian case, there are no big differences between EKF and HF estimations. Both filters can fairly estimate the robot path and landmarks location as seen on Fig.4. Both estimations show familiar results and capable of building the map although HF shows a slightly higher covariances than KF. In gaussian noise case, KF achieved better result than HF. Nevertheless, HF is proved to be useful for SLAM problem and can support the theoretical analysis in previous section.

Both Figs.6-7 demonstrates the results of applying case 2 of different initial covariance into the system(bigger initial covariance with random noise observation). Both filters shows diverse estimations. But it is observable HF perform better in this case. KF estimation is erroneous even though the landmark initial covariance is small. Even though in Fig.7 HF covariance is bigger than KF, this result proves that HF is
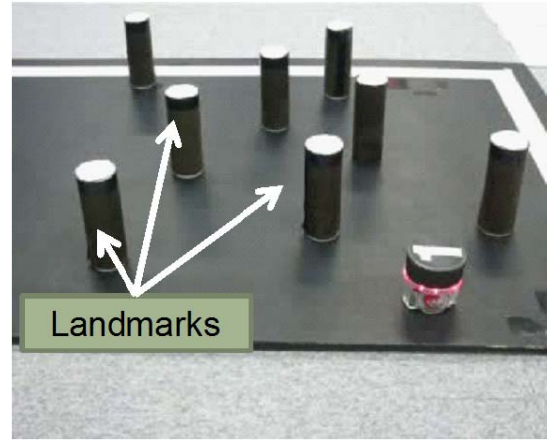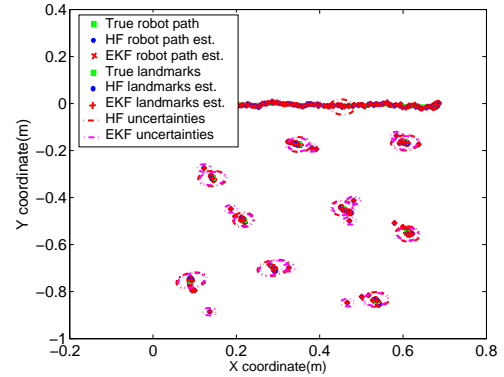
converging thus proves *Theorem 2* and more robust than KF for robot with bigger uncertainties and under non-gaussian noise environment. We inspect that HF is preferred for an environment that has an unknown noise characteristics than KF. An investigation for *Lemma 1* and *Theorem 1* are also required. In the case of $\gamma^2 << R$, HF estimation is erroneous(Fig.8). Thus, inherently causing unreliable estimations of both robot and landmarks location. Therefore unlike KF, in HF, $\gamma^2 >> R$ must be satisfied to achieve a desired results and performance.

We summarized that HF is appropriate to be applied in a system where model changes unpredictably and has an unknown properties. Furthermore, HF is suitable when the noises are known in some bounded energy. We has described in the proposed theorems that γ will be bigger if the measurement and process noise is bigger. Unfortunately, HF is more complicated than KF and the designer must carefully choose the noise parameters and initial covariance.

VI. CONCLUSIONS

HF may need further improvement and development to achieve more stable and motivating results in SLAM. Even so, HF is capable to approximate linear and non-linear
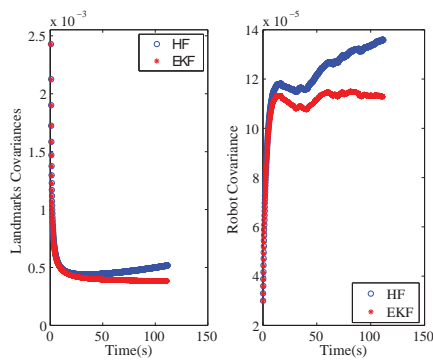
Fig. 5. Robot and landmarks covariances under zero mean Gaussian noises
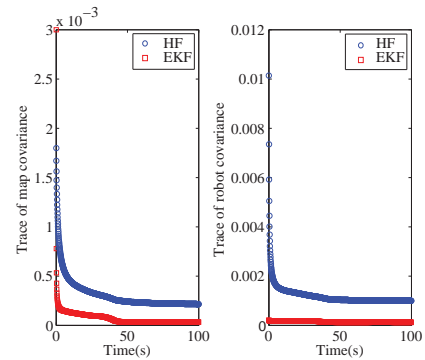

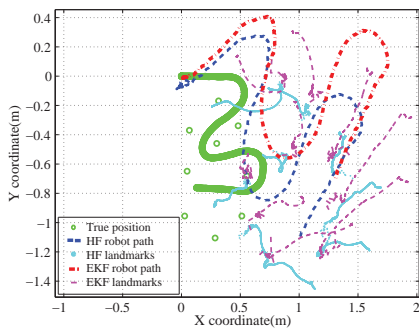Fig. 7. Robot and landmarks covariances under bigger initial uncertainties

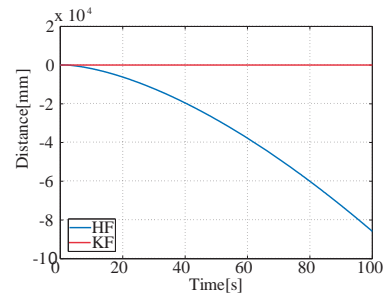Fig. 6. Constructed map under bigger initial uncertainties

Fig. 8. The landmarks estimation when robot is stationary:Effect of $R >> \gamma$

system that has wide coverage and variety of noise and has proven to be useful for SLAM problem especially for bigger initial state covariance in an unknown noise statistics. However, the designer should consider appropriate level of weighting noises of Q, and R to achieve certain level of performance as HF is more sensitive and complicated to the design parameters.

A. Acknowledgements

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